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A quick method of estimating absolute biomass from a time series of relative biomass and catch data

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MOTIVATION FOR THE METHOD

Consider two hypothetical examples. First, population A decreases by 50% during a period when 1000 tonnes are caught. Second, population B decreases by 10% during the same period with the same catch. If this was the only information available, which population do you think is smaller? Probably population A is smaller since it decreased more rapidly which implies that the catch is a larger fraction of population A than of population B. It is possible that population B is smaller, but it did not decrease as rapidly because it was more productive (i.e. growth plus recruitment minus natural mortality was higher).

The point is that population biomass can be estimated from the rate of change in relative biomass in response to fishing. The estimate also depends on the rate of production. De Lury (1947) first used this approach. There are numerous modifications (Allen 1966, Collie and Sissenwine 1983). A model which expresses the idea mathematically is given below.

THE MODEL

The rate of change in population biomass is,

$$\frac{dB}{dt} = (P-F)B \tag{1}$$

where P is the constant instantaneous production rate (reflecting growth, recruitment, and natural mortality) and F is the constant instantaneous fishing mortality rate. The solution to this differential equation is,

$$B_t = B_0 \; e^{(P-F)t} \tag{2}$$

where B_t is biomass at time t and B_0 is biomass at time 0. The catch (C) is,

$$C = \frac{FB_0}{(F-P)} \; (1-e^{(P-F)t}) \tag{3}$$

These are the fundamental equations of fish population dynamics (with slight modifications). They can be rearranged and combined to estimate biomass. Let $r = B_t/B_0$ (i.e. the proportional change in biomass), then

$$P-F = \frac{1}{t} \mathrm{LOG}_e(r) \tag{4}$$

$$F = P-\frac{1}{t} \mathrm{LOG}_e(r) \tag{5}$$

These equations can be substituted into Equation (3), resulting in:

$$C = \frac{(P-v)B_0}{-v} (1-r) \tag{6}$$

where $v = (1/t)\ \mathrm{LOG}_e\ (r)$. By rearranging terms,

$$B_0 = \frac{vC}{(v-P)\ (1-r)} \tag{7}$$

$$B_t = \frac{vrC}{(v-P)\ (1-r)} \tag{8}$$

DISCUSSION

Equation 8 can be used to estimate biomass at time t as a function of the catch during the preceding time period from 0 to t, the biomass ratio (r), instantaneous rate of change in biomass (v), and the instantaneous production rate (P). For example, if 1000 tonnes are caught during a period of 1 year, and trawl surveys indicate that relative biomass decreased by 20% ($r = 0.8$), and the production rate is 10% per year ($P = 0.1$), then B_t is estimated as 2762 tonnes. But if $P = 0.0$, $B_t = 4000$.

Equation 8 requires an estimate of the catch. Errors in the catch estimate have a proportional effect on the biomass estimates. If the catch estimate used in the examples given above was believed to underestimate the actual catch by 50% (i.e. the actual catch was believed to be 1500 tonnes instead of 1000 tonnes), then the biomass estimates should be adjusted upward by a factor of 1.5 (to 4143 tonnes and 6000 tonnes, respectively).

The biomass ratio can be estimated from a time series of either standardised trawl survey relative biomass indices, or standardised catch per unit effort data from the fishery. In both cases it is assumed that the index is proportional to biomass. Since relative abundance indices are subject to several sources of error, it is better to apply Equation 8 to a series of indices over several years. This also averages out some of the year-to-year variability in the instantaneous rate of production (P).

It is important to remember that the model on which Equation 8 is based assumes that F and P are constant during the period from time 0 to t. This means that the catch rate (i.e. per unit time) should change during the period in proportion to changes in B. For example, if B decreases by 50% over several years, the catch in the last year should be about 50% of the catch in the first year in order not to violate a model assumption. Fortunately, the results are not very sensitive to this assumption unless catch is heavily concentrated during a brief period toward the beginning or end of the time interval.

It is more difficult to evaluate the validity of the assumption of constant productivity. In theory, P should be inversely related to B; i.e. increases in P compensate for reductions in population size. But in reality, the relationship between P and B is poorly defined. While it is unlikely that P is constant during any time period, it is reasonable, for the purpose of a quick biomass estimate, to use an average value of P.

Unfortunately, P is the most difficult parameter of the model to estimate in most circumstances. Fisheries scientists usually select a target fishing mortality rate which is intended to maximize the long-term average yield. Frequently, $F_{0.1}$ is the target. Implicitly, this means that the average production equals $F_{0.1}$. In fact, many fish stocks have been able to sustain a slightly higher fishing mortality rate (sometimes much higher). Since $F_{0.1}$ is usually somewhat higher than M, P = M is probably a somewhat conservative rule of thumb. But, P is variable. Changes in average size of fish are a useful clue about anomalies in P.

If the average size of fish increases, this means that P is lower than usual. P might be as low as -M for a population of old fish (i.e. zero growth) during a period of total recruitment failure, but this would be an extreme situation.

It is more difficult to interpret a decrease in average size. If there is a sudden decrease (particularly when the relative abundance index increases) it probably means unusually good recruitment, and P above average. On the other hand, a gradual decrease in average size during a period of decreasing relative abundance might occur even with average P. This happens when the fishing mortality rate is increasing (indicated by the ratio of catch to relative biomass increasing).

In addition to the average size of the fish in the catch, the abundance of pre-recruits in trawl surveys is an important clue about productivity. If pre-recruits are abundant, productivity can be expected to be above average in the future, and vice versa.

In fact, if there are data on the size and age composition of the catch, and there are indices of both recruited biomass and pre-recruit abundance, a variety of methods are available to take advantage of all data. But these are not quick methods, and they are beyond the scope of this note.

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