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Assessments of the stock of southern blue whiting (*Micromesistius australis*) from the Campbell Island Rise

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This series documents the scientific basis for stock assessments and fisheries management advice in New Zealand. It addresses the issues of the day in the current legislative context and in the time frames required. The documents it contains are not intended as definitive statements on the subjects addressed but rather as progress reports on ongoing investigations.

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EXECUTIVE SUMMARY

1. Assessments of the stock of southern blue whiting from the Campbell Island Rise using data up to 1993 are conducted using Laurec-Shepherd *ad hoc* tuned Virtual Population Analysis (VPA) and the sensitivity of the results to several of the specifications of the base-case is examined.

2. The fits of the models are very poor – well below the levels which should provide reliable results. The precision of the estimates is poor, but not as poor as some of the diagnostics of the VPA would suggest.

3. The results obtained using the "Lowestoft" variant of *ad hoc* tuned VPA were discarded because this variant predicts that more fish in the plus-group die than is expected given the estimates of natural and fishing mortality.

4. Some of the management related quantities are very sensitive to the value assumed for the rate of natural mortality. The estimates of the $F_{status-quo}$ strategy TAC on the other hand, are fairly insensitive to this value. These estimates are, however, sensitive to the catch for 1994 and the number of years used to define $F_{status-quo}$.

5. Another age-structured assessment method, ADAPT, suggests that the fully selected fishing mortality in 1993 was very low (0.01 yr^{-1}), whereas the base case *ad hoc* tuned VPA assessment suggests that this mortality is about 0.1 yr^{-1}.

6. The base case estimate of Maximum Sustainable Yield (*MSY*) is 15 200 t.

7. The base case estimate of the $F_{status-quo}$ strategy TAC for 1994 is 8 400 t and the estimate of the $F_{0.1}$ strategy TAC for 1994 is 26 300 t.

8. The base-case estimates of the $F_{status-quo}$ strategy TAC for 1995 range between 7 800 and 15 700 t, and the estimates of the $F_{0.1}$ strategy TAC for 1995 range between 25 600 and 27 200 t.

9. The quality of VPA-based assessments will improve once the catch rate time series increases, and if other sources of relative abundance information become available.

1. INTRODUCTION

Hanchet (1993) assessed the three stocks of southern blue whiting off New Zealand (Campbell Rise, Pukaki Rise, and Bounty Platform) using data up to 1992. Due to the paucity of catch age-composition data, it was impossible to apply virtual population analysis (VPA) to assess the Pukaki Rise and Bounty Platform stocks.

The most recent year (1992) terminal fishing mortalities for the Campbell Rise stock VPA were tuned using the Laurec-Shepherd tuning algorithm (Pope & Shepherd 1985). Unfortunately, the relationship between fishing mortality and fishing effort for this stock is weak, and the catch rate for 1992 (which was derived from data for Soviet super-Atlantic vessels) is questionable (Hanchet 1993). These factors resulted in the 1993 Fishery Assessment Plenary (Annala 1993) rejecting the results of the VPA analyses and suggesting instead an analysis based on examining the sensitivity of back-projection results over a plausible range for the fully selected fishing mortality for 1992 (Hanchet 1993).

Ad hoc tuned VPA is based on several assumptions. Hanchet (1993) examined the sensitivity of the VPA results to alternative prescriptions for the placement of the plus-group and the length of the catch rate time series. This paper examines sensitivity further by considering the implications of changes to these specifications in addition to changes to the method used to tune the oldest-age terminal fishing mortalities, the effect of systematic changes in selectivity over older ages, and changes to the number of ages used when tuning the oldest-age terminal fishing mortalities. The sensitivity of the results to the choice of assessment method is examined by applying ADAPT as well as *ad hoc* tuned VPA.

The catch-at-age and relative abundance information used in the analyses are listed in Table 1, and Table 2 provides information on mass-at-age. Note that the fishing effort information in Table 1 is based on the results of a general linear model standardisation (e.g. Gavaris 1980, Kimura 1981) unlike the fishing effort information considered by Hanchet (1993). Table 3 lists the specifications of the base case analysis and the 13 sensitivity tests. The base case assessment is identical to that of Hanchet (1993) except that fishing mortality is assumed to occur over the final 5% of the year only.

2. METHODS

Hanchet (1993) noted that the relationship between fishing effort and fishing mortality is weak, but did not estimate the variances of the estimated quantities. In this paper, a (conditioned) parametric bootstrap approach (*see* Appendix I) is applied for this purpose. The *ad hoc* tuned VPA and ADAPT methods, as applied here, are described in Appendices II and III. Apart from some small modifications, they are identical to the methods applied routinely at ICES, CAFSAC, and ICCAT (amongst others). The following sections list these modifications.

2.1 Population dynamics model

The population dynamics and TAC determination formulae have been adjusted to take account of the assumption that fishing mortality occurs over the final 5% of the year only. The sensitivity of the results to two assumptions regarding the dynamics of the plus-group (*see* Equations II.2 to II.4) is examined. The difference between the two methods is that the first (the Lowestoft method) is based on the assumption that the fishing mortality on the plus-group is equal to that on the next lowest age-class, whereas the other (the ICCAT method) is based on ensuring that the historic plus-group dynamics follow the rules used when projecting the population forward (i.e., the number in the plus-group in year $y+1$ must equal the number in the plus-group in year y plus the number in the next lowest age-class in that year, less the fishing and natural mortality during year y).

2.2 Estimation of recruitment

The estimates of recruitment, particularly those for the most recent years, produced by VPA are often very imprecise. In order to reduce the variance of some of the recruitment estimates for the most recent years, they have been "shrunk" by pooling the VPA estimates and the estimate of the geometric mean recruitment using inverse-variance-weighting (*see* Equation II.10). The geometric mean recruitment (also used to predict the strength of the 2 year old age class in all future (i.e., 1994, 1995, etc.) years) is calculated over the years 1982 to 1990. The recruitments for years 1991 to 1993 are "shrunk" to the geometric mean recruitment (i.e., $k = 3$ when applying Equation II.10). The selection of $k = 3$ is based on an examination of the variances of the VPA estimates of recruitment.

2.3 TAC determination

Information on catch-at-age is available until 1993. Unfortunately, estimates of TAC according to the $F_{0.1}$ and $F_{status-quo}$ strategies are needed for 1995 as well as 1994. In order to estimate TACs for 1995, it is necessary to specify the fishing mortalities for 1994 (the VPA produces estimates of the numbers at the start of 1994: *see* Equation II.15 and surrounding text). Four alternative prescriptions have been considered in order to capture the range of possibilities.

a) The fishing mortalities-at-age for 1994 are the same as those for 1993 (the $F_{status-quo}$ assumption). The numbers-at-age at the start of 1995 (year $t+2$) are thus given by Equation II.16.
b) The 1994 catch is set to 7 000 t and the catch equation (*see* Equation II.17) is solved for the fully selected fishing mortality during 1994 (*see* Equation II.18).
c) As for b) except that the 1994 catch is assumed to be 11 000 t.
d) As for b) except that the 1994 catch is assumed to be 15 000 t.

A further problem arises when estimating $F_{status-quo}$ strategy TACs for 1995 because it is not obvious what fishing mortalities-at-age to use for 1995. The two possibilities are: those for 1993 (option 1), and those for 1994 (option 2). Naturally, if option a) above is used to specify the fishing mortalities for 1994, the two options are identical.

In summary, therefore, 2 TACs are estimated for 1994 (1 for each harvesting strategy) and 12 TACs for 1995 (4 for the $F_{0.1}$ strategy – 1 for each of prescriptions a) – d), and 8 for the $F_{status-quo}$ strategy – 1 for each combination of prescriptions a) – d) and options 1) and 2)). The 4 $F_{0.1}$ strategy TACs for 1995 are distinguished by adding the letter of the prescription used to specify the 1994 fishing mortality to the acronym for the TAC. The 8 $F_{status-quo}$ strategy TACs for 1995 are distinguished by adding the letter of the prescription used to specify the 1994 fishing mortalities and the number of the option used to specify $F_{status-quo}$ to the acronym for the TAC.

3. RESULTS

Table 4 provides the two base case (one for the ICCAT method and one for the Lowestoft method) estimates of the numbers and fishing mortality-at-age matrices and also gives the standard deviations of the residuals of the fishing mortality *vs.* effort relationships (σ_a – *see* Equation II.6) for each base case assessment. Results are shown for both methods of handling the plus-group to indicate the sensitivity of the point estimates and the measures of uncertainty to this specification.

Table 5 lists point estimates, bootstrap means, standard errors of logarithms (*SELs*), and percentile method 90% confidence intervals for 22 management related quantities (acronyms in parenthesis) for the ICCAT method *ad hoc* tuned VPA base-case assessment.

a) The exploitable biomass during 1982 (B^e_{1982}).
b) The exploitable biomass during 1986 (B^e_{1986}).
c) The exploitable biomass during 1993 (B^e_{1993}).
d) The exploitable biomass during 1993 expressed as a fraction of that in 1982 (B^e_{1993} / B^e_{1982}).
e) The exploitable biomass during 1993 expressed as a fraction of the corresponding unexploited equilibrium level (B^e_{1993} / K^e).
f) The exploitable biomass during 1993 expressed as a fraction of the exploitable biomass at which *MSY* is achieved (B^e_{1993} / B^e_{MSY}).
g) The exploitable biomass at which *MSY* is achieved (B_{MSY}).
h) The average fishing mortality on ages 4–10 in the most-recent-year (F_{bar}).
i) *MSY* – the maximum sustainable yield (*MSY*).
j) The $F_{0.1}$ strategy TAC for 1994 (94 $F_{0.1}$ TAC).
k) The $F_{status-quo}$ strategy TAC for 1994 (94 F_{sq} TAC).
l) The four $F_{0.1}$ strategy TACs for 1995 (95 $F_{0.1}$ TAC).
m) The eight $F_{status-quo}$ strategy TACs for 1995 (95 F_{sq} TAC).

[*SELs* are provided instead of *CVs* because they are less sensitive to skew distributions, and because the *SEL* of an estimate is approximately equal to its *CV* if the distribution of the estimate is log-normal – another reason for considering *SELs* is the inherent positivity of the estimates of the management related quantities.] Table 6 indicates the sensitivity of the estimates of the $F_{0.1}$ and $F_{status-quo}$ TACs for 1995 to doubling the estimate of the number of 2-year-olds at the start of 1993. This year-class was selected as information from acoustic surveys (S.M. Hatchet, MAF Fisheries Greta Point, pers. comm.) suggests that the 1991 year class will be strong even though this is not yet evident in the VPA. Table 7 examines the sensitivity of the $F_{status-quo}$ strategy TACs for

1995 to the definition of $F_{status-quo}$. Results are shown for definitions of $F_{status-quo}$ of the form:

$$F_{status-quo,a} = \frac{1}{w} \sum_{y=t-w+1}^{t} \hat{F}_{y,a} \qquad (1)$$

The results in Tables 5 and 6 correspond to the choice $w = 1$; Table 7 examines the sensitivity of the results to the choices $w = 2$ and $w = 3$.

Table 8 shows the sensitivity of the results of the *ad hoc* tuned VPA to changes to the specifications of the stock assessment, in the form of percentage changes in the values of 8 of the 22 management related quantities. The $F_{0.1}$ TAC in this Table is "95 $F_{0.1}$ TAC (c)" and the F_{sq} TAC in this Table is "95 F_{sq} TAC (1,c)". The sensitivities of the other TAC estimates are qualitatively similar to those for the two presented.

Figure 1 shows the base case estimates of exploitable biomass and their 90% confidence intervals, as well as the corresponding estimates for the two sensitivity tests which involve changing the number of years in the assessment. Results are shown in Figures 1(a) and (b) for the Lowestoft and ICCAT methods of handling the plus-group respectively.

4. DISCUSSION

4.1 The reliability of the results

Almost all of the estimates obtained in these analyses are very poorly determined. The estimates of the standard deviations of the residuals of the fishing mortality *vs.* fishing effort relationships (σ_a, – *see* Table 4 and Equation II.6) are often used as indicators of reliability. Conventional wisdom suggests that reliable estimation should occur when σ_a is less than 0.4 and that the estimates provided by *ad hoc* tuned VPA should be interpreted with considerable caution when a number of the σ_as are greater than 0.6. None of the estimates of σ_a in Table 4 are less than 0.4 and the bulk are greater than 0.6 (all for the ICCAT method and all but three for the Lowestoft method). This means that the estimates of fishing mortality-at-age for the most recent year are very poorly determined. The estimates of the numbers-at-age at the start of 1993, and hence the biomass during 1993, are poorly determined as they are related inversely to the fishing mortalities-at-age for 1993.

The estimated *SEL*s in Table 5 are not as high as might have been expected from the values of σ_a given in Table 4. However, the 90% confidence intervals are extremely skewed (*see* Figure 1). The variances reported in this document are negatively biased because age correlation effects are ignored when generating the pseudo fishing mortality matrices: *see* Butterworth *et al.* (1990) for details. It is unclear whether this bias is substantial. Butterworth *et al.* (1990) attempted to take age correlation effects into account in the assessment of the northern Namibian stock of Cape horse mackerel and found them to be small.

A further cause for concern are the retrospective patterns in exploitable biomass evident in Figure 1. It is clear from this figure that the results are extremely dependent upon the effort for the most recent year.

The results for ADAPT are not presented here as this assessment method suggests that the fishing mortality during 1993 was very small and that the changes in catch rate are attributable to fluctuations in year class strength only. Note, however, that the fit of the ADAPT model, as measured by the standard deviation of the log-residuals, is as good as that of the *ad hoc* tuned VPA.

Bearing in mind the caveats raised in this section, the results in Tables 4 to 8 will now be discussed. The base case results will be discussed first, followed by those for the various sensitivity tests.

4.2 Base case assessments

Qualitatively, the results of the ICCAT and Lowestoft methods are similar (*see* Table 4), but there are some important differences between them. As expected, the differences are associated with the dynamics of the plus-group. The Lowestoft method results (*see* Table 4a) suggest that the number in the plus-group dropped markedly between 1982 and 1984. However, this appears to be an artifact of the method used to handle the plus-group because the combination of fishing and natural mortality is unable to reduce the numbers in the plus-group by the extent indicated by the assessment. This suggests that the results for the Lowestoft method should be accorded less weight than those for the ICCAT method. All the remaining results in this paper were computed using the ICCAT method.

The base case assessment suggests that the 1988 year class is extremely strong, that the number in the plus-group has dropped markedly, and that fishing mortality in the most-recent-year is small (< 0.1 yr^{-1}). The resource is estimated currently to be almost equal to its 1982 size. The primary reason for this is that the strong 1988 year class is now 6 years old and is counter balancing the reduction in the numbers of 11+ animals. There appears to be almost no chance that the resource has been depleted to below the biomass at which *MSY* is achieved (the lower 90% confidence limit for the ratio B^e_{1993} / B^e_{MSY} exceeds 4). Note that this last conclusion is dependent on the assumption that recruitment is independent of spawner biomass.

The point estimate of *MSY* is 15 200 t. The point estimate of the $F_{0.1}$ strategy TAC for 1994 is 24 600 t, although its 90% confidence interval includes all values between 15 600 and 54 300 t. The $F_{status-quo}$ strategy TAC for 1994 is much lower at 8 400 t. The $F_{0.1}$ strategy TAC for 1995 ranges between 25 600 and 27 200 t depending on the method used to handle the plus-group and the prescription used to specify the fishing mortalities for 1994. All of these estimates have *SELs* about 0.4 which is large by fisheries standards. The $F_{status-quo}$ strategy TACs for 1995 range between 7 800 and 15 700 t. If the fishing mortalities for 1993 are used to estimate the $F_{status-quo}$ strategy TAC for 1995, this TAC is largely independent of the prescription used to specify the fishing mortalities for 1994. However, this is not so if the fishing mortalities for 1994 are used to calculate the $F_{status-quo}$ strategy TACs for 1995. The latter TACs are determined with considerably lower *SELs* than the former, because the catch for 1994 is fixed. This leads to less variability in the estimates of $F_{status-quo}$ by age.

4.3 Sensitivity analyses

The effect of the 1991 year class being twice as large as the VPA estimate is to increase the $F_{0.1}$ strategy TAC for 1995 by roughly 3 000 t. The effect on the $F_{status-quo}$ strategy TAC for 1995 is small if the fishing mortalities for 1994 are used to calculate the $F_{status-quo}$ strategy TACs for 1995, and roughly 900 t if the fishing mortalities for 1993 are used to calculate the $F_{status-quo}$ strategy TACs for 1995 (*see* Table 6). The *SELs* in Table 6 are marginally smaller than the corresponding values in Table 5.

The effect on the $F_{status-quo}$ strategy TACs for 1995 of changing the value of w (*see* Equation 1) is quite marked. As the value of w is increased from 1 to 3, the $F_{status-quo}$ strategy TACs for 1995 virtually double. This suggests that the $F_{status-quo}$ strategy TACs are sensitive to this specification.

The point estimate of the $F_{status-quo}$ strategy TAC is least sensitive to the changes to the specifications of the VPA (*see* Table 8). On the other hand, the estimates of B^e_{1993}, B^e_{1993} / K^e, B^e_{1993} / B^e_{MSY}, F_{bar}, and the $F_{0.1}$ strategy TAC for 1995 are all extremely sensitive to the value assumed for the rate of natural mortality.

5. CONCLUDING REMARKS

The results of this assessment must be treated with considerable caution. The *ad hoc* tuned VPA method is based on the assumption that the effort for the most recent year is **exact**. The effort for 1993 is markedly lower than that for 1991 and 1992. This leads the *ad hoc* tuned VPA to the conclusion that fishing mortality was very low during 1993.

The fit of the model to the data is poor and the *SELs* for all the estimated quantities high. Furthermore, except for the $F_{status-quo}$ strategy TACs, these estimates are all sensitive to the value assumed for the rate of natural mortality. The $F_{status-quo}$ strategy TAC for 1994 is reasonably well defined at around 8 500 t. However, this quantity is sensitive to the number of years included when calculating $F_{status-quo}$. Including the most recent 3 years when estimating $F_{status-quo}$ leads to a doubling of the TAC. The $F_{status-quo}$ strategy TAC for 1995 is sensitive to the catch during 1994 as well as to the number of years included when calculating $F_{status-quo}$.

The reliability of the results of analyses such as those conducted in this document will be improved as the length of the catch rate time series increases and if other sources of information on relative abundance (perhaps based on trawl / acoustic surveys) become available.

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Table 1 : Catch-at-age and fishing effort data for the Campbell Island Rise stock of southern blue whiting (source, Hanchet 1994). Catch units are '000 fish

Age	Year											
	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
1	0	0	4	5	2	2	2	0	3	2	3	0
2	2 753	1 280	45	296	860	1 338	2 464	379	997	727	1 886	577
3	7 076	10 350	1 129	172	3 438	3 619	4 716	9 030	4 328	29 971	1 122	6 616
4	1 143	7 687	5 333	1 675	409	4 744	5 788	6 981	14 923	6 983	12 539	761
5	479	549	6 100	3 718	1 876	463	4 230	5 372	2 752	6 353	4 035	5 327
6	256	428	780	3 037	6 223	1 191	118	4 969	1 934	1 709	3 790	1 336
7	1 231	336	554	186	5 925	4 293	1 596	391	1 767	1 132	423	1 866
8	197	1 154	471	103	775	3 454	6 059	2 224	278	556	467	260
9	169	268	1 143	181	438	390	3 012	6 446	527	244	325	219
10	401	0	365	395	635	299	187	4 316	1 383	432	57	131
11	513	596	106	195	676	287	436	1 333	1 188	704	194	58
12	224	429	149	108	215	388	174	239	248	440	128	36
13	134	0	557	498	337	157	440	564	51	200	149	172
14	274	136	210	361	354	262	172	321	243	92	94	54
15	422	472	263	283	479	275	357	350	229	58	0	19
16	206	448	198	132	284	224	435	56	32	95	0	6
17	1 126	572	191	267	314	144	232	370	0	34	44	5
18	1 228	1 808	267	24	200	112	57	291	145	64	27	20
19	450	710	687	192	687	77	26	0	64	55	0	39
Effort base case	-	-	-	-	15 252	20 411	30 095	52 015	35 836	56 686	59 597	13 239
Effort delta-log	-	-	-	-	15 252	20 259	29 945	51 592	34 862	56 913	68 971	13 728

Table 2 : Mass-at-age data for the Campbell Island Rise stock of southern blue whiting (source, Hanchet 1994). The mass for a fish in the plus-group (when this group is not 19+) in year y, is a weighted mean of the masses-at-age for all ages represented by the plus-group. The weight assigned to each age when calculating this weighted mean is equal to the catch of the age concerned

Age (a)	Mass (kg)
2	0.193
3	0.326
4	0.448
5	0.550
6	0.630
7	0.690
8	0.735
9	0.763
10	0.785
11	0.802
12	0.815
13	0.821
14	0.825
15	0.827
16	0.828
17	0.830
18	0.831
19+	0.832

Table 3 : Specifications of the base case assessment and the sensitivity tests

Acronym	Natural mortality (yr^{-1})	Plus-group age (m)	Selectivity slope (γ)	Tuning parameter (p)
base-case	0.2	11	0	6
$M = 0.1yr^{-1}$	0.1	11	0	6
$M = 0.15yr^{-1}$	0.15	11	0	6
$M = 0.25yr^{-1}$	0.25	11	0	6
$M = 0.3yr^{-1}$	0.3	11	0	6
$m = 8$	0.2	8	0	6
$m = 16$	0.2	16	0	6
$\gamma = 0.1$	0.2	11	0.1	6
$\gamma = -0.1$	0.2	11	-0.1	6
$p = 2$	0.2	11	0	2
$p = 4$	0.2	11	0	4
Delta-lognormal	0.2	11	0	6
1982–92	0.2	11	0	6
1982–91	0.2	11	0	6

Table 4 : Base case estimates of the number- and fishing mortality-at-age matrices, and of the values of $\hat{\sigma}_a$ obtained from the *ad hoc* tuned VPA. Results are shown for the two alternative methods of handling the plus-group

(a) Lowestoft method - numbers-at-age

	Age									
Year	2	3	4	5	6	7	8	9	10	11+
1982	160735	136999	11449	10114	12939	21718	7108	0	9423	111718
1983	43338	128860	105124	8237	7804	10338	16557	5623	0	97755
1984	8852	34209	95204	78421	6197	5964	8130	12407	4337	43920
1985	57125	7203	26885	72641	58137	4298	4331	6188	9021	59562
1986	51834	46475	5726	20345	55774	44577	3334	3444	4886	40227
1987	45536	41582	34630	4281	14790	39473	30602	1959	2384	23288
1988	154871	35950	30444	23633	3044	10924	28047	21618	1216	17909
1989	88077	124346	24742	19167	15141	2375	7356	16935	14703	12444
1990	235535	71735	92821	13313	10350	7455	1556	3811	7455	12543
1991	27707	191848	54425	61150	8162	6550	4346	997	2596	10581
1992	163584	21961	127255	37612	43745	4982	4236	3005	574	6401
1993	85019	132055	16864	91712	26780	32044	3658	3004	2137	6671
1994	-	69034	101535	13050	69787	20596	24379	2737	2241	6674

Fishing mortality-at-age

	Age									
Year	2	3	4	5	6	7	8	9	10	11+
1982	0.0210	0.0648	0.1293	0.0593	0.0243	0.0714	0.0343	0.0000	0.0531	0.0531
1983	0.0366	0.1027	0.0931	0.0845	0.0690	0.0403	0.0885	0.0597	0.0725	0.0725
1984	0.0062	0.0409	0.0705	0.0993	0.1660	0.1198	0.0730	0.1187	0.1079	0.1079
1985	0.0063	0.0295	0.0787	0.0642	0.0656	0.0540	0.0293	0.0362	0.0547	0.0547
1986	0.0204	0.0942	0.0908	0.1189	0.1457	0.1761	0.3319	0.1679	0.1719	0.1719
1987	0.0364	0.1118	0.1821	0.1409	0.1030	0.1417	0.1475	0.2770	0.1654	0.1654
1988	0.0195	0.1736	0.2627	0.2452	0.0482	0.1954	0.3045	0.1855	0.2069	0.2069
1989	0.0052	0.0924	0.4198	0.4163	0.5086	0.2232	0.4577	0.6205	0.4410	0.4410
1990	0.0052	0.0761	0.2173	0.2892	0.2575	0.3397	0.2448	0.1840	0.2554	0.2554
1991	0.0324	0.2105	0.1695	0.1350	0.2936	0.2357	0.1690	0.3528	0.2259	0.2259
1992	0.0141	0.0641	0.1275	0.1397	0.1113	0.1089	0.1438	0.1409	0.1287	0.1287
1993	0.0083	0.0628	0.0564	0.0732	0.0625	0.0734	0.0903	0.0928	0.0774	0.0774
$\hat{\sigma}_a$	1.1525	0.6245	0.5586	0.5836	0.7353	0.6168	0.7455	0.6099	0.5573	N/A

(Table 4 Continued)

(b) ICCAT method - numbers-at-age

Year	Age									
	2	3	4	5	6	7	8	9	10	11+
1982	149406	126725	10349	9297	8425	9720	32106	0	1659	58448
1983	37051	119585	96713	7336	7135	6643	6734	26090	0	44083
1984	14485	29062	87610	71533	5460	5416	5104	4365	21094	30496
1985	54830	11815	22670	66423	52497	3694	3883	3710	2437	38198
1986	50771	44596	9502	16894	50684	39959	2840	3077	2858	30282
1987	44951	40712	33092	7373	11965	35305	26821	1554	2083	21300
1988	153823	35471	29732	22373	5575	8612	24634	18523	884	15941
1989	88835	123487	24350	18585	14110	4447	5463	14142	12169	10849
1990	241335	72355	92119	12992	9873	6610	3252	2261	5169	10920
1991	31495	196596	54933	60575	7900	6159	3654	2386	1327	9482
1992	151122	25063	131143	38029	43274	4768	3917	2439	1711	6668
1993	83535	121850	19404	94896	27121	31659	3483	2742	1673	6170
1994	-	67819	93180	15129	72394	20875	24064	2593	2027	5885

Fishing mortality-at-age

Year	Age									
	2	3	4	5	6	7	8	9	10	11+
1982	0.0226	0.0703	0.1441	0.0647	0.0376	0.1671	0.0075	0.0000	0.3476	0.1041
1983	0.0429	0.1111	0.1016	0.0953	0.0757	0.0634	0.2335	0.0126	0.5303	0.1685
1984	0.0038	0.0484	0.0769	0.1094	0.1907	0.1327	0.1189	0.3829	0.0213	0.1593
1985	0.0066	0.0179	0.0941	0.0704	0.0729	0.0631	0.0328	0.0611	0.2193	0.0866
1986	0.0208	0.0984	0.0537	0.1450	0.1616	0.1987	0.4028	0.1899	0.3147	0.2354
1987	0.0368	0.1143	0.1914	0.0794	0.1289	0.1599	0.1702	0.3637	0.1916	0.1823
1988	0.0197	0.1762	0.2699	0.2610	0.0261	0.2551	0.3550	0.2201	0.2970	0.2357
1989	0.0052	0.0931	0.4282	0.4326	0.5583	0.1130	0.6822	0.8065	0.5636	0.5260
1990	0.0050	0.0755	0.2192	0.2975	0.2718	0.3927	0.1097	0.3330	0.3931	0.2996
1991	0.0284	0.2049	0.1678	0.1363	0.3050	0.2527	0.2044	0.1327	0.5034	0.2557
1992	0.0153	0.0559	0.1235	0.1380	0.1125	0.1141	0.1565	0.1766	0.0413	0.1232
1993	0.0084	0.0683	0.0488	0.0707	0.0617	0.0743	0.0951	0.1021	0.1000	0.0840
$\hat{\sigma}_a$	1.1607	0.6509	0.6049	0.6315	0.9459	0.7691	0.8700	0.7950	1.0898	N/A

Table 5 : Base case management variable estimates, bootstrap means, standard errors of logarithms and percentile method 90% confidence intervals obtained from the ICCAT method *ad hoc* tuned VPA

Quantity	Estimate	Bootstrap mean	*SEL*	90% confidence interval	
B^e_{1982}	84247	82377	0.040	77786	88485
B^e_{1986}	63916	63428	0.074	57603	72979
B^e_{1993}	82869	96701	0.386	51463	183038
B^e_{1993} / B^e_{1982}	0.984	1.174	0.375	0.647	2.155
B^e_{1993} / K^e	0.712	0.805	0.281	0.517	1.286
B^e_{1993} / B^e_{MSY}	5.667	6.402	0.281	4.115	10.23
B_{MSY}	14622	15105	0.141	12224	19184
F_{bar}	0.079	0.106	0.414	0.057	0.209
MSY	15230	15733	0.141	12733	19982
94 $F_{0.1}$ TAC	24645	28641	0.372	15608	54343
94 F_{sq} TAC	8413	12958	0.391	7678	25785
95 $F_{0.1}$ TAC (a)	26937	29872	0.391	15741	56768
95 $F_{0.1}$ TAC (b)	27217	31221	0.388	16283	59122
95 $F_{0.1}$ TAC (c)	26426	30357	0.399	15472	58262
95 $F_{0.1}$ TAC (d)	25636	29490	0.411	14661	57401
95 F_{sq} TAC (1,a)	9255	13373	0.384	7920	27070
95 F_{sq} TAC (1,b)	9352	13877	0.390	8204	28714
95 F_{sq} TAC (1,c)	9079	13496	0.395	8001	28185
95 F_{sq} TAC (1,d)	8806	13113	0.400	7733	27871
95 F_{sq} TAC (2,a)	9255	13373	0.384	7920	27070
95 F_{sq} TAC (2,b)	7785	7689	0.110	6675	9313
95 F_{sq} TAC (2,c)	11859	11731	0.114	10071	14125
95 F_{sq} TAC (2,d)	15662	15518	0.119	13207	18817

Table 6 : Base case $F_{0.1}$ and $F_{status-quo}$ strategy TAC estimates for 1995, bootstrap means, standard errors of logarithms and percentile method 90% confidence intervals obtained from the ICCAT method *ad hoc* tuned VPA. These TACs were calculated by doubling the strength of the 1991 year-class

Quantity	Estimate	Bootstrap mean	*SEL*	90% confidence interval	
$F_{0.1}$ TAC (a)	29934	32822	0.386	17404	60911
$F_{0.1}$ TAC (b)	30437	34474	0.382	18386	63499
$F_{0.1}$ TAC (c)	29657	33619	0.392	17618	62736
$F_{0.1}$ TAC (d)	28877	32761	0.403	16851	61972
F_{sq} TAC (1,a)	10133	14396	0.383	8523	29291
F_{sq} TAC (1,b)	10303	15016	0.389	8838	31014
F_{sq} TAC (1,c)	10039	14646	0.393	8477	30567
F_{sq} TAC (1,d)	9774	14274	0.399	8283	30120
F_{sq} TAC (2,a)	10133	14396	0.383	8523	29291
F_{sq} TAC (2,b)	7534	7502	0.112	6501	9270
F_{sq} TAC (2,c)	11525	11486	0.114	9896	14092
F_{sq} TAC (2,d)	15289	15248	0.118	13021	18572

Table 7 : The sensitivity of the base case $F_{status-quo}$ strategy TACs for 1995 to the catch assumed for 1994, and the number of years used when calculating $F_{status-quo}$ fishing mortalities-at-age (w). The figures given in parenthesis are the standard errors of the logarithms of the estimates

1994 Catch (t)	Number of years used to calculate $F_{status-quo}$ (w)		
	1	2	3
$F_{status-quo}$	9 255 (0.384)	12 008 (0.339)	15 183 (0.321)
7 000	9 352 (0.390)	12 412 (0.359)	16 342 (0.343)
11 000	9 079 (0.395)	12 042 (0.365)	15 865 (0.350)
15 000	8 806 (0.400)	11 672 (0.372)	15 388 (0.358)

Table 8 : Relative changes in the point estimates of eight of the 22 management related quantities (expressed as percentages) for 11 of the sensitivity tests for the ICCAT method *ad hoc* tuned VPA assessment. Figures with an absolute value larger than 20% are shown in bold type and those with an absolute value larger than 10% are shown in italics

Acronym	Quantity							
	B^e_{1993}	B^e_{1993} / B^e_{1982}	B^e_{1993} / K^e	B^e_{1993} / B^e_{MSY}	F_{bar}	MSY	$F_{0.1}$ TAC	F_{sq} TAC
$M = 0.1\text{yr}^{-1}$	**-27.89**	*16.87*	**-53.79**	**-63.10**	**44.30**	**-20.70**	**-55.28**	4.23
$M = 0.15\text{yr}^{-1}$	-16.24	8.33	**-27.53**	**-37.13**	**21.52**	*-14.33*	**-31.62**	2.02
$M = 0.25\text{yr}^{-1}$	**24.47**	-7.93	**28.51**	**53.91**	**-21.52**	**26.05**	**47.23**	-1.92
$M = 0.3\text{yr}^{-1}$	**65.66**	*-15.24*	**57.16**	**94.57**	**-41.77**	**73.88**	**127.01**	-3.79
$\gamma = -0.1$	-1.77	0.81	9.83	-0.34	1.27	-0.71	6.43	-0.35
$\gamma = 0.1$	-6.11	-1.22	-7.16	0.26	-2.53	0.07	-5.34	0.08
$p = 2$	3.03	-1.02	-0.28	0.35	0.00	-2.07	-1.79	-0.46
$p = 4$	2.38	-1.22	-2.81	-0.04	2.53	-3.50	-4.73	-1.21
$m = 8$	-2.95	-8.64	4.63	-2.51	-18.99	*-15.34*	-7.77	0.38
$m = 16$	**-47.15**	5.08	4.35	7.75	-6.33	*-10.85*	-6.04	-5.46
delta lognormal	-3.85	-3.86	-2.39	-2.38	3.80	-1.47	-4.31	-0.03

Appendix I : Estimating *CVs* and confidence intervals

The estimates of standard error (*SE*) and thence coefficient of variation (*CV*) are obtained by means of the (conditioned) parametric bootstrap procedure (Efron 1982, 1985), and estimates of 90% confidence intervals by applying the percentile method (Efron 1981, Buckland 1984).

Estimation of standard errors and coefficients of variation

The (conditioned) parametric bootstrap variance-estimation procedure is illustrated by an example: estimating the standard error of the quantity $\{\alpha + \beta x_1\}$, where α and β are obtained by means of a linear regression of $\{y_i : i = 1,\ldots,k\}$ on $\{x_i : i = 1,\ldots,k\}$. There is little need to use a bootstrap procedure in this instance because, under the assumption that errors are independent and identically normally distributed, the variance required can be estimated analytically using normal distribution theory. This particular example has been chosen purely to simplify explanations.

More generally, let $\hat{Q}$ be the quantity for which an estimate of standard error is required. Assume that $\hat{Q}$ is calculated from the set of estimated model parameters $\underline{\hat{P}}$, using the function ϕ, i.e.

$$\hat{Q} = \phi(\underline{\hat{P}}) \tag{I.1}$$

where, for the linear regression example, $\underline{\hat{P}} = \{\hat{\alpha}, \hat{\beta}\}$, and $\phi(\alpha, \beta)$ is $\{\alpha + \beta x_1\}$.

Now, let $\underline{\theta}$ be the estimator of $\underline{\hat{P}}$, given a set of data **X**, i.e.:

$$\underline{\hat{P}} = \underline{\theta}(\mathbf{X}) \tag{I.2}$$

In the example under consideration, **X** is the set $\{x_i : i = 1,..,k;\ y_i : i = 1,..,k\}$, and the estimator $\underline{\theta}$ is:

$$\underline{\theta} = \left[\frac{\sum x_i^2 \sum y_i - \sum x_i y_i \sum x_i}{k \sum x_i^2 - \sum x_i \sum x_i} \quad \frac{\sum x_i y_i - \sum x_i \sum y_i / k}{\sum x_i^2 - \sum x_i \sum x_i / k} \right] \tag{I.3}$$

Finally, let $\hat{\mathrm{D}}(\mathbf{X})$ be an estimate of the distribution of the observed data points **X**. The various forms of the bootstrap (Efron 1981, 1982, 1985, 1987) differ as to how $\hat{\mathrm{D}}(\mathbf{X})$ is constructed. The (conditioned) parametric bootstrap is "conditioned" in the sense that the values of the independent variables (i.e. $\{x_i : i = 1, .. ,k\}$ in the example considered) are assumed to be fixed. The procedure is "parametric" because, in order to construct $\hat{\mathrm{D}}(\mathbf{X})$, it is assumed that the distributions of the dependent variables are identical to those assumed when fitting the model. For the regression example considered here, these distributions are:

$$y_i \sim N(\alpha + \beta x_i; \sigma^2)$$

Note that the conditioning is equivalently written: $x_i \sim N(x_i; 0)$.

The (conditioned) parametric bootstrap variance-estimation procedure then estimates the standard error of $\hat{Q}$ as follows: a large number (U_{max} - taken here to be 500) of random bootstrap samples $\{\mathbf{X}^{U}: U = 1,\ldots,U_{max}\}$ are generated, and the set $\{\hat{Q}^1, \hat{Q}^2, \ldots, \hat{Q}^{U_{max}}\}$ where $\hat{Q}^U = \phi[\underline{\theta}(\mathbf{X}^U)]$ is computed. For the regression example, the random bootstrap samples are generated as follows:

$$x_i^U \sim \hat{D}(x_i) = x_i$$
$$y_i^U \sim \hat{D}(y_i) = N(\hat{\alpha} + \hat{\beta} x_i; \hat{\sigma}^2)$$

The variance of $\hat{Q}$ is then estimated by:

$$\mathrm{VA\hat{R}}(Q) = \frac{1}{U_{max} - 1} \sum_{U=1}^{U_{max}} [\hat{Q}^U - Q_{(.)}]^2 \qquad \text{(I.4)}$$

where $Q_{(.)}$ is the mean of the $\hat{Q}^U$s.

In order to implement the (conditioned) parametric bootstrap, it is necessary to specify the functions ϕ, $\underline{\theta}$ and the distribution $\hat{\mathrm{D}}(\mathbf{X})$. As ϕ is usually trivially defined, and $\underline{\theta}$ is the estimator itself, the descriptions of the implementations of the (conditioned) parametric bootstrap for each approach detail only the distribution $\hat{\mathrm{D}}(\mathbf{X})$, i.e., how the random bootstrap samples are generated.

Confidence intervals

The percentile method (Efron 1981, Buckland 1984) is used to estimate confidence intervals. The estimate of the confidence interval is computed from the set of bootstrap estimates after sorting them into ascending order. If N Monte-Carlo trials are performed, the 90% confidence interval for $\hat{Q}$ is estimated (to sufficient accuracy for the purposes required here) by $[\hat{Q}^U_{0.05N}, \hat{Q}^U_{0.95N}]$, with $\hat{Q}^U_{0.05N}$ and $\hat{Q}^U_{0.95N}$ obtained by linear interpolation within the ordered set.

Appendix II : The *ad hoc* tuned VPA estimator (Butterworth *et al.* 1990, Punt and Butterworth 1991, Punt 1994)

To obtain the estimates of the fishing mortality ($F_{y,a}$) and numbers-at-age ($N_{y,a}$) matrices required to compute $F_{status-quo}$ catches (see below), the standard VPA back-calculations for each cohort, together with the selected tuning algorithms, are applied iteratively until convergence takes place. The techniques used to tune the terminal fishing mortalities are described first, followed by the methods used to estimate the management related quantities. Finally, the method of generating the bootstrap replicates is detailed.

The VPA back-calculations

The VPA back-calculation process is used to calculate the entire numbers-at-age matrix (N) from the numbers-at-age for the oldest-age (age *m*, taken to be a plus-group) and the most-recent-year (year *t*). Fishing mortality is assumed to take place in the final 5% of the year only (V. Haist, Pacific Biological Station, Nanaimo, pers. comm.) so that the number of animals of age *a* caught during year *y* ($C_{y,a}$) is given by the equation:

$$C_{y,a} = \frac{F_{y,a}\, N_{y,a} e^{-0.95 M_a}}{F_{y,a} + 0.05 M_a}[1 - \exp\{-(F_{y,a} + 0.05 M_a)\}] \qquad \text{(II.1)}$$

where $F_{y,a}$ is the fishing mortality on animals of age *a* during year *y*, and

M_a is the instantaneous rate of natural mortality on animals of age *a* (assumed to independent of year).

For $a < m-1$, $N_{y,a}$ is calculated from $N_{y+1,a+1}$ using the equation:

$$N_{y,a} = N_{y+1,a+1} e^{(M_a + F_{y,a})} \qquad \text{(II.2)}$$

Two alternative approaches (the Lowestoft method and the ICCAT method) are used to calculate the number of animals in the plus-group. The Lowestoft method involves assuming that the fishing mortality on the plus-group is equal to that on the next lowest age so that, given values for $C_{y,m}$ and $F_{y,m}$ (= $F_{y,m-1}$), Equation (II.1) can be solved for the number in the plus-group ($N_{y,m}$).

The ICCAT method is based on the equation governing the dynamics of the plus-group:

$$N_{y+1,m} = N_{y,m-1}\, e^{-(M_{m-1} + F_{y,m-1})} + N_{y,m}\, e^{-(M_m + F_{y,m})} \qquad \text{(II.3)}$$

Given values for $F_{y,m-1}$, $F_{y,m}$ and the catch-at-age matrix, values for $N_{y,m-1}$ and $N_{y,m}$ can be calculated using the formulae:

$$N_{y,m-1} = C_{y,m-1}\, e^{0.95M_{m-1}}\, (0.05M_{m-1} + F_{y,m-1}) / [F_{y,m-1}\,(1 - e^{-(0.05M_{m-1}+F_{y,m-1})})] \qquad (\text{II}.4a)$$

$$N_{y,m} = C_{y,m}\, e^{0.95M_m}\, (0.05M_m + F_{y,m}) / [F_{y,m}\,(1 - e^{-(0.05M_m+F_{y,m})})] \qquad (\text{II}.4b)$$

To calculate $N_{y,m-1}$ and $N_{y,m}$ for $y < t$, Equation (II.3) is solved for $F_{y,m-1}$ given a value of $N_{y+1,m}$. A value for $N_{y+1,m}$ will either have been calculated from the results of the previous iteration of the most recent year tuning algorithm (for $y = t-1$) or estimated by applying the algorithm described below to year $y+1$. The calculation procedure is as follows.

i) Guess $F_{y,m-1}$.
ii) Calculate $F_{y,m}$ (see Equations II.5a and II.5b). If $p > 1$, the values of $F_{y,m-p}$ to $F_{y,m-2}$ will already have been calculated.
iii) Calculate $N_{y,m}$ and $N_{y,m-1}$ from Equation (II.4).
iv) Substitute all four values ($F_{y,m-1}, F_{y,m}, N_{y,m-1}$ and $N_{y,m}$) into the right hand side of Equation (II.3), and compare the result to the left hand side, which is known.
v) Iterate (i) - (iv) until Equation (II.3) is satisfied.

Values for $F_{t,m-1}$ and $F_{t,m}$ (and hence for $N_{t,m-1}$ and $N_{t,m}$) are obtained from the tuning algorithm for the most recent year (see Equations II.6), followed by application of Equations II.4a and II.4b

Tuning procedure

The algorithm used to tune the oldest-age terminal fishing mortalities is based on the assumption that the age-specific selectivity function changes over the oldest $p+1$ ages at a rate γ per year. Two alternative error structures are considered.

The equation specifying the fishing mortality on the plus-group as a function of those on the p younger ages is:

$$\hat{F}_{y,m} = \left[\prod_{a=m-p}^{m-1} F_{y,a}\, e^{(m-a)\gamma} \right]^{1/p} \qquad y = 1,\ldots,t \qquad (\text{II}.5\text{a})$$

if the error about the relationship is assumed to be log-normal, or

$$\hat{F}_{y,m} = \tfrac{1}{p}\left[\sum_{a=m-p}^{m-1} \left(F_{y,a}[1 + (m-a)\gamma] \right) \right] \qquad y = 1,\ldots,t \qquad (\text{II}.5\text{b})$$

if this error is assumed to be normal.

Note that fixing $\gamma = 0$ corresponds to the special case where age-specific selectivity is assumed to be flat (constant) at large ages. Note also that age *m* is replaced by age *m*-1 when applying the Lowestoft Method for calculating the number of animals in the plus-group (because $F_{y,m}$ is set equal to $F_{y,m-1}$ for this method).

The method applied to tune the most recent year terminal fishing mortalities is the Laurec-Shepherd tuning algorithm (Pope & Shepherd 1985):

$$\hat{F}_{t,a} = \overline{q_a}\, E_t \qquad a = 2,3,\dots,m-1 \tag{II.6}$$

where $\overline{q_a} = \left[\prod_{y=1}^{t-1} (F_{y,a} / E_y)\right]^{1/(t-1)}$

$\hat{\sigma}_a^2 = \frac{1}{t-2}\sum_{y=1}^{t-1}\left[\ell n(F_{y,a} / E_y) - \ell n\overline{q_a}\right]^2$

$\overline{q_a}$ is the catchability coefficient for age a, and

E_y is the effort for year y.

$F_{0.n}$ and $F_{status-quo}$ strategy estimation

In the estimation of quantities for the $F_{0.n}$ and $F_{status-quo}$ harvesting strategies, the overall calculation involves the following three steps: specification of the selectivity pattern, estimation of $F_{0.n}$, and prediction of recent and future recruitments. Each of these steps is described in turn.

SPECIFICATION OF THE SELECTIVITY PATTERN S_a

Specification of the S's is achieved using the formula (see Equation II.6):

$$\hat{S}_a = \overline{q_a} / \overline{q_m} \quad \text{where} \quad \overline{q_m} = \hat{F}_{t,m} / E_t \tag{II.7}$$

ESTIMATION OF $F_{0.n}$

The equilibrium yield-per-recruit to fishing mortality (F) relationship is given by the equation:

$$Y/R = \sum_{a=2}^{m} w_{a+1/2}\, S_a\, F\, N_a\, e^{-0.95M_a} \frac{1-\exp\{-(0.05M_a + S_aF)\}}{0.05M_a + S_aF} \tag{II.8}$$

where $w_{a+1/2}$ is the mass of a fish of age a in the middle of the fishing season,

$N_2 = 1$ (animals aged 0 and 1 years are ignored because they are not taken by the fishery : for convenience therefore only ages of 2 and above are considered), and

$$N_a = \begin{cases} N_{a-1}\, e^{-(M_{a-1}+S_{a-1}F)} & a < m \\ \dfrac{N_{m-1}\, e^{-(M_{m-1}+S_{m-1}F)}}{1-e^{-(M_m+S_mF)}} & a = m \end{cases}.$$

The value of $F_{0.n}$ is obtained numerically by solving the equation:

$$\left.\frac{d(Y/R)}{dF}\right|_{F=F_{0.n}} = 0.n \left.\frac{d(Y/R)}{dF}\right|_{F=0} \tag{II.9}$$

The F_{MSY} strategy (sometimes referred to as F_{MSYR} or F_{MAX}) corresponds to setting n=0.

PREDICTION OF RECENT AND FUTURE RECRUITMENTS

The variances of the recruitment estimates provided by VPA are usually very large for the last few years of a time series (*see*, for example, Butterworth *et al.* 1990). Therefore, methods of *TAC* estimation which use such estimates are likely to provide relatively imprecise results. Thus, the last k recruitment values (i.e., corresponding to years $t-k+1$ to t) are replaced with inverse-variance-weighted averages of the VPA-estimated recruitment for that year and the estimated mean recruitment.

$$N_{t-i,2} = \exp\left[\{\ell n(\hat{N}_{t-i,2})w_i^N + \ell n(\hat{R})w_R\} / (w_i^N + w_R)\right] \tag{II.10}$$

where w_i^N is the inverse of the variance of $\ell n(\hat{N}_{t-i,2})$,

w_R is $(\hat{\sigma}_R^2)^{-1}$,

$$\hat{R} = \left[\prod_{y=1}^{t-k} \hat{N}_{y,2}\right]^{1/(t-k)}$$

$$\hat{\sigma}_R^2 = \frac{1}{t-k-1}\sum_{y=1}^{t-k}\left[\ell n\hat{N}_{y,2} - \ell n\hat{R}\right]^2$$

This approach, suggested to the author by J.G. Shepherd (Fisheries Laboratory, Lowestoft, pers. comm. to D.S. Butterworth) is commonly referred to as "shrinkage". In general, it results in estimates of recruitment for the last few years of the series which are similar to the geometric mean recruitment. The balance of the numbers-at-age matrix is then computed by forward-projecting these alternative recruitment estimates under the historic catches-at-age.

This method of specifying recent recruitments requires estimates of the variances of $\ell n(\hat{N}_{t-i,2})$. These are obtained by a Monte-Carlo bootstrap variance-estimation procedure. This procedure involves the generation of a large number of bootstrap replicate fishing-mortality matrices, which are conditioned on the assumption that the catch-at-age matrix and the estimates of natural mortality-at-age are exact. These matrices follow from the standard VPA back-calculations given alternative sets of terminal fishing mortalities, which are defined as follows. The symbol U is used to denote the Uth bootstrap replicate assessment.

MOST RECENT YEAR

$$F_{t,a}^{U} = \hat{F}_{t,a}\exp(\varepsilon_{t,a}^{U}) \quad a = 2,3,\ldots,m-1 \qquad \varepsilon_{t,a}^{U} \sim N(0;\sigma_a'^2) \tag{II.11}$$

where $\hat{\sigma}_a'^2 = \left[\frac{1}{t-1}+1\right]\hat{\sigma}_a^2$

(the formula for $\hat{\sigma}_a^2$ is given under Equation II.6)

OLDEST-AGE

$$F^{U}_{y,m} = \hat{F}_{y,m}\exp(\varepsilon^{U}_{y,m}) \quad y = 1,2,\ldots,t \qquad \varepsilon^{U}_{y,m} \sim N(0;\hat{\sigma}'^2) \tag{II.12a}$$

if a log-normal error structure is assumed, or

$$F^{U}_{y,m} = \hat{F}_{y,m} + \varepsilon^{U}_{y,m} \quad y = 1,2,\ldots,t \qquad \varepsilon^{U}_{y,m} \sim N(0;\hat{\sigma}'^2) \tag{II.12b}$$

if a normal error structure is assumed.

where $\hat{\sigma}'^2 = [\frac{1}{p} + 1]\hat{\sigma}^2$, and

$$\hat{\sigma}^2 = \begin{cases} \dfrac{1}{(t-1)(p-1)} \displaystyle\sum_{y=1}^{t-1} \sum_{a=m-p}^{m-1} \left[\ell n\hat{F}_{y,a} + \gamma\,(m-a) - \ell n\hat{F}_{y,m}\right]^2 & \text{if lognormal} \\ \dfrac{1}{(t-1)(p-1)} \displaystyle\sum_{y=1}^{t-1} \sum_{a=m-p}^{m-1} \left[\hat{F}_{y,a}[1+\gamma\,(m-a)] - \hat{F}_{y,m}\right]^2 & \text{if normal} \end{cases} \tag{II.13}$$

The recruitments for years $y = t+1, t+2$ are estimated to be the geometric mean of past recruitments:

$$\hat{R} = \left[\prod_{y=1}^{t-k} \hat{N}_{y,2}\right]^{1/(t-k)} \tag{II.14}$$

Note that the complete period $y = 1$ to $t - k$ has been used to estimate mean recruitment, which assumes no relationship between recruitment and spawning biomass. No such relationship is immediately evident for the stock considered here over the period for which catch-at-age data are available. Therefore the approach seems appropriate here.

Using this estimate ($\hat{R}$) for $\hat{N}_{t+1,2}$, and the formula:

$$\hat{N}_{t+1,a+1} \;=\; \hat{N}_{t,a}\, e^{-(M_a + \hat{F}_{t,a})} \tag{II.15}$$

the numbers-at-age present at the start of year $t+1$ can be estimated.

To estimate a TAC for year $t+2$, it is necessary to project the age-structure at the start of year $t+1$ (see Equation II.15) to the start of year $t+2$. Two alternative methods are considered to achieve this. The first method involves assuming that the fishing mortalities-at-age during year $t+1$ are the same as those during year t, i.e. the numbers-at-age at the start of year $t+2$ are given by the formula:

$$\hat{N}_{t+2,a} = \begin{cases} \hat{R} & a=2 \\ \hat{N}_{t+1,a-1}\, e^{-(M_{a-1}+\hat{F}_{t,a-1})} & 3 \le a \le m-1 \\ \hat{N}_{t+1,m-1}\, e^{-(M_{m-1}+\hat{F}_{t,m-1})} + \hat{N}_{t+1,m}\, e^{-(M_m+\hat{F}_{t,m})} & a=m \end{cases} \tag{II.16}$$

The second method for projecting the numbers-at-age at the start of year $t+1$ to the start of year $t+2$ involves specifying the catch (rather than the fishing mortality) for year $t+1$. This method thus involves solving the following equation for the fully selected fishing mortality during year $t+1$:

$$\tilde{C}_{t+1} = \sum_{a=2}^{m} w_{a+1/2}\, S_a\, F_{t+1}\, \hat{N}_{t+1,a}\, e^{-0.95M_a} \frac{1-\exp\{-(0.05M_a + S_a F_{t+1})\}}{0.05M_a + S_a F_{t+1}} \tag{II.17}$$

where $\tilde{C}_{t+1}$ is the specified catch for year $t+1$.

The numbers-at-age at the start of year $t+2$ are then given by the formula:

$$\hat{N}_{t+2,a} = \begin{cases} \hat{R} & a=2 \\ \hat{N}_{t+1,a-1}\, e^{-(M_{a-1}+S_{a-1}F_{t+1})} & 3 \le a \le m-1 \\ \hat{N}_{t+1,m-1}\, e^{-(M_{m-1}+S_{m-1}F_{t+1})} + \hat{N}_{t+1,m}\, e^{-(M_m+S_m F_{t+1})} & a=m \end{cases} \tag{II.18}$$

The $F_{0.1}$ harvesting strategy TAC for year $t+2$ is given by:

$$\mathrm{TAC}_{sq} = \sum_{a=2}^{m} w_{a+1/2} S_a F_{0.1} \hat{N}_{t+2,a}\, e^{-0.95M_a} \frac{1-\exp\{-(0.05M_a + S_a F_{0.1})\}}{0.05M_a + S_a F_{0.1}} \tag{II.19}$$

The version of the $F_{status-quo}$ harvesting strategy (Pope 1983, Pope & Gray 1983) considered here aims to keep fishing mortality at its current level. A problem arises here because it is not obvious what the "current level" is in this case. The two options considered here are: the fishing mortalities-at-age for year t and those for year $t+1$, i.e.:

$$\mathrm{TAC}_{sq} = \sum_{a=2}^{m} w_{a+1/2}\, \hat{F}_{t,a}\, \hat{N}_{t+2,a}\, e^{-0.95M_a} \frac{1-\exp\{-(0.05M_a + \hat{F}_{t,a})\}}{0.05M_a + \hat{F}_{t,a}} \tag{II.20a}$$

or

$$\mathrm{TAC}_{sq} = \sum_{a=2}^{m} w_{a+1/2}\, S_a\, F_{t+1}\, \hat{N}_{t+2,a}\, e^{-0.95M_a} \frac{1-\exp\{-(0.05M_a + S_a F_{t+1})\}}{0.05M_a + S_a F_{t+1}} \tag{II.20b}$$

Other management quantities

THE $F_{0.n}$ HARVESTING STRATEGY TARGET BIOMASS LEVEL

The $F_{0.n}$ harvesting strategy target biomass level (taken as a midseason value) is given by the formula:

$$B^e_{0.n} = \sum_{a=2}^{m} w_{a+1/2}\, S_a\, \hat{R} \exp\{-\sum_{a'=2}^{a-1}(M_{a'} + S_{a'} F_{0.n})\}\, e^{-0.95 M_a}\, e^{-(0.05 M_a + S_a F_{0.n})/2} \qquad \text{(II.21)}$$

The resource's average unexploited equilibrium biomass, K^e, is obtained from Equation (II.21) by replacing $F_{0.n}$ by 0.

THE $F_{0.n}$ HARVESTING STRATEGY TARGET CATCH LEVEL

The $F_{0.n}$ harvesting strategy target catch level is given by the formula:

$$C_{0.n} = \sum_{a=2}^{m} \frac{w_{a+1/2}\, S_a\, F_{0.n}}{0.05 M_a + S_a\, F_{0.n}}\, \hat{R}\, \exp\{-\sum_{a'=2}^{a-1}(M_{a'} + S_{a'}\, F_{0.n})\}\, e^{-0.95 M_a}\, (1 - e^{-(0.05 M_a + S_a F_{0.n})}) \qquad \text{(II.22)}$$

The maximum sustainable yield, *MSY*, is obtained from Equation (II.22) by setting $F_{0.n}$ to F_{MSY}. This evaluation of *MSY* assumes recruitment to be independent of spawning biomass.

THE EXPLOITABLE BIOMASS SERIES

The exploitable biomass during year y (taken as the midseason value) is defined as:

$$B^e_y = \sum_{a=2}^{m} w_{a+1/2}\, S_a\, N_{y,a}\, e^{-0.95 M_a}\, e^{-(0.05 M_a + F_{y,a})/2} \qquad \text{(II.23)}$$

Generating the bootstrap replicates of the management quantities

Each bootstrap simulation involves generating pseudo fishing mortality matrices, applying the VPA back-calculation procedure and calculating the estimates of the quantities of interest. Equations (II.11) and (II.12) are used to generate the pseudo fishing mortality matrices.

When estimating the variance of $F_{status-quo}$ and $F_{0.1}$ harvesting strategy TACs, only the estimates of future recruitment and numbers-at-age in the final year are assumed to be stochastic. The numbers-at-age data are generated as a by-product of the variance estimation procedures described above, while the alternative future recruitments are obtained from the equation:

$$N^{\mathrm{U}}_{t+j,2} = R^{\mathrm{U}} \exp(\varepsilon^R) \qquad j = 1,2,\ldots \qquad \varepsilon^R \sim N(0;\hat{\sigma}^2_R) \qquad \text{(II.24)}$$

where $R^{\mathrm{U}} = \left[\prod_{y=1}^{t-k} N^{\mathrm{U}}_{y,2}\right]^{1/(t-k)}$

Note that $\hat{\sigma}^2_R$ (*see* Equation II.10) has not been adjusted to allow for error in the estimation of the mean in the same way that, for example, $\hat{\sigma}_a$ is amended to $\hat{\sigma}'_a$ in Equation (II.11). The reason is that the Monte Carlo variation in the $N^{\mathrm{U}}_{y,2}$ used to calculate R^{U} allows for this source of variability.

The bootstrap values for the revised estimates of historical recruitment are generated as follows:

$$N^{\mathrm{U}}_{t-i,2} = \exp[\{\ell n(N^{\mathrm{U}}_{t-i,2})w_i^N + \ell n(R^{\mathrm{U}}e^{\varepsilon^{\mathrm{U}}})w_R\}/(w_i^N + w_R)] \qquad \text{(II.25)}$$

where w_i^N is the inverse of the variance of $\ell n(\hat{N}_{t-i,2})$,
w_R is $(\hat{\sigma}_R^2)^{-1}$, and
ε^U from $N(0;\hat{\sigma}_R^2)$.

The inclusion of the ε^U factor in Equation (II.25) is motivated by an appeal to the limiting case of $w_i^N \to 0$. Although the estimate of $N^{\mathrm{U}}_{t-i,2}$ is then given by R^{U}, because no other information is available, the variance calculations must take account of the fact that this is just the geometric mean of the distribution from which the actual recruitment in year $(t-i)$ would eventuate. The ε^U factor is included to account for this additional variability.

Appendix III: The ADAPT Methodology
(Gavaris 1988, Butterworth and Punt 1992, Punt 1994)

The ADAPT assessment method involves estimating the numbers-at-age matrix ($N_{y,a}$) by maximizing a likelihood function involving observed and model-predicted indices of abundance. The model parameters are (usually) a subset of the numbers-at-age at the start of year $t+1$, where t is the most recent year for which catch-at-age data are available.

This appendix details first the process of calculating the N-matrix from the vector of parameters. It then specifies the likelihood function to be maximised. Finally, the manner in which the bootstrap N-matrices are generated is described. Once the N-matrix has been estimated, the management quantities are calculated as for *ad hoc* tuned VPA (Appendix II), except that the recent recruitments are not replaced by the inverse-variance-weighted average of the estimate and the geometric mean recruitment.

Calculation of the N-matrix from the vector of parameters

The numbers-at-age at the start of year t are calculated from the parameter vector, which contains estimates of some of the numbers-at-age at the start of year $t+1$. If $N_{t+1,a+1}$ is one of the parameters, then the standard VPA back-calculation procedure (suitably modified if age $a+1$ is the plus-group) is used to calculate $N_{t,a}$.

If $N_{t+1,a+1}$ is not one of the parameters, then the selectivity function for all fleets and gears combined, and the fishing mortality for a "reference" age $a = ref$, is used to calculate $F_{t,a}$, from which $N_{t,a}$ and $N_{t+1,a+1}$ are obtained. $F_{t,a}$ is calculated using the equation:

$$F_{t,a} = S_a^T \, F_{t,\mathrm{ref}} / S_{\mathrm{ref}}^T \qquad \text{(III.1)}$$

where S_a^T is the selectivity of the gear (all fleets combined) on fish of age a,
ref is the reference age, and
$F_{t,ref}$ has been obtained from input parameter $N_{t+1,ref+1}$ using the VPA back-calculation procedure.

The remainder of the N-matrix (i.e. $N_{y,a} : 1 \le y < t; 2 \le a \le m$) is calculated using the standard VPA back-calculation procedure, together with a prescription to define fishing mortalities for the oldest age (*see* Equation II.5), modified to handle a plus-group.

The likelihood function

In addition to CPUE data, the estimator can also use biomass survey data to estimate the model parameters. The approach here assumes that the biomass survey estimates are relative indices of abundance, and that they are log-normally distributed about their expected values. Estimates of the parameter values are then obtained by maximizing the appropriate likelihood function:

$$L=\prod_{y}\exp\{-\hat{v}_y^2/(2\hat{\sigma}_v^2)\}/(\sqrt{2\pi}\hat{\sigma}_v)\ \prod_{j}\exp\{-\hat{v}_j^2/[2(\hat{\sigma}^S)^2]\}/(\sqrt{2\pi}\hat{\sigma}^S) \qquad \text{(III.2)}$$

where the first product is over all years (y) for which CPUE data are available,
the second product is over all available biomass survey estimates (j),
$\hat{v}_j=\ell n B_j^S-\ell n(\hat{\chi}\ \hat{B}_j^e)$,

χ is a constant of proportionality (the relative bias of the biomass survey estimates - assumed to be the same for 1991 and 1992),

B_j^S is the jth biomass survey estimate,

B_y^e is the exploitable biomass during year y:

$$B_y^e=\sum_{a=1}^{m} w_{a+1/2}\,S_{y,a}\,N_{y,a}\,e^{-0.95M_a}\,e^{-(0.05M_a+F_{y,a})/2}$$
$$S_{y,a}=F_{y,a}/\max(F_{y,a'}:a'=1,2,\ldots,m) \qquad \text{(III.3)}$$

$\hat{v}_y=\ell n(C/E)_y-\ell n(C\hat{/}E)_y$

$(C\hat{/}E)_y=\hat{q}\,\hat{B}_y^e$

q is the catchability coefficient,

$\hat{\sigma}_v^2=\sum\hat{v}_y^2/\sum 1$. No attempt is made to adjust this estimate for bias, and

$(\hat{\sigma}^S)^2=\sum\hat{v}_y^2/\sum 1$. No attempt is made to adjust this estimate for bias.

By taking natural logarithms, changing signs and eliminating constants, maximising Equation (III.2) can be shown to be equivalent to minimizing:

$$-\ell nL=\sum_{y}[\ell n(\hat{\sigma}_v)]+\sum_{j}[\ell n(\hat{\sigma}^S)] \qquad \text{(III.4)}$$

where the first summation is over all years (y) for which CPUE data are available, and the second summation is over all available biomass survey estimates (j).

Generating the bootstrap replicates of the management quantities

Each bootstrap simulation involves generating "pseudo" (bootstrap replicate) catch rate and biomass survey data based on the fit of the model (III.4) to the actual data, and then fitting the model to these pseudo-data. The pseudo-data are generated as follows:

$$(C/E)_y^{U}=(C\hat{/}E)_y\,e^{v_y^{U}} \qquad v_y^{U}\sim N(0;\hat{\sigma}_v^2) \qquad \text{(III.5}a\text{)}$$

$$B_j^{S,U}=\hat{\chi}\,\hat{B}_j^e\,e^{v_j^{S,U}} \qquad v_j^{S,U}\sim N(0;(\hat{\sigma}^S)^2) \qquad \text{(III.5}b\text{)}$$

where $(C\hat{/}E)_y$ is the estimate of catch rate in year y obtained by applying the estimator to the actual data,

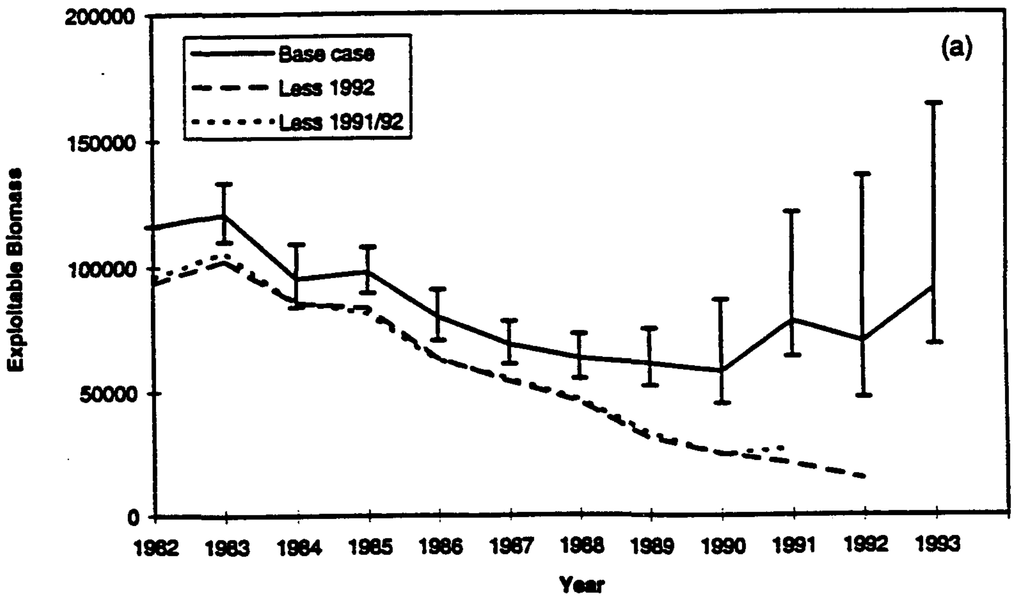
$(C/E)_y^{U}$ is the catch rate for year y in artificial data set U,

$\hat{B}_j^e$ is the model estimate corresponding to the jth survey biomass estimate, obtained by applying the estimator to the actual data,

$\hat{\chi}$ is the model estimate of the bias of survey biomass estimates, and

$B_j^{S,\mathrm{U}}$ is the jth survey biomass estimate in artificial data set U.

It is necessary to allow for the fact that selectivity for the oldest-age will fluctuate about its predicted value. The formula used for *ad hoc* tuned VPA (Equation II.12) is applied for this purpose.



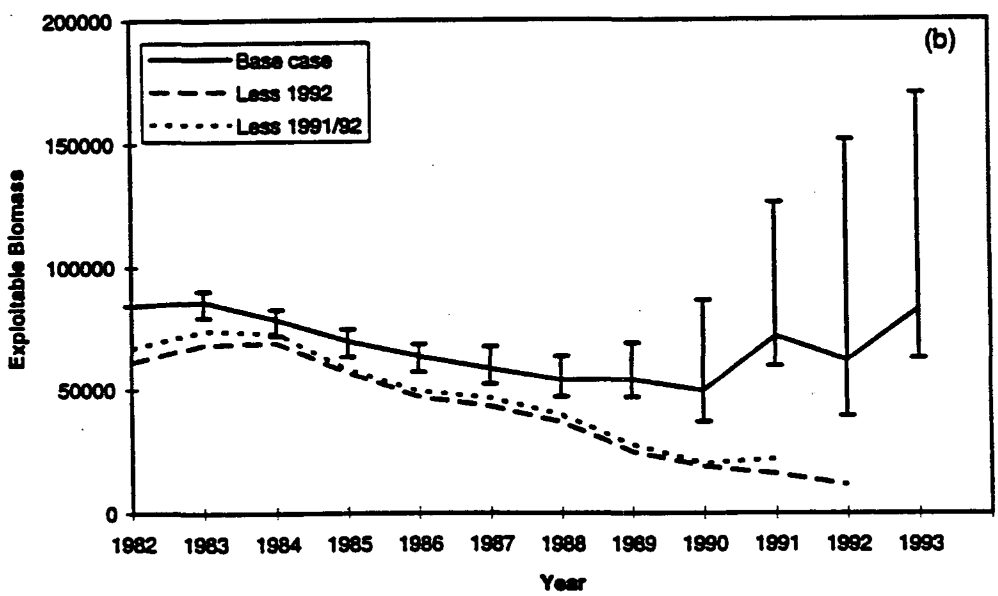


Figure 1 : Base case trajectories of exploitable biomass, with bootstrap 90% confidence intervals using (a) the Lowestoft and (b) the ICCAT methods of handling the plus-group. The exploitable biomass trajectories obtained from assessments based on data up to 1991 and 1992 respectively are also shown