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An evaluation of alternative stock reduction estimators of virgin biomass and of the information content of various research survey scenarios

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1. Executive summary

Two simulation studies were undertaken to determine which of two stock reduction estimators of virgin biomass was better, and to determine appropriate survey frequencies for relative biomass indices and absolute biomass indices.

Both studies used similar models and methods. An "operating model" (an assumed model of reality, which includes statistical assumptions about the observations) was used to generate simulated observations. For each of a number of estimation methods, a population model (identical in structure to that of the operating model) was used in a fitting procedure to estimate virgin biomass and the proportionality constant for a relative abundance time series. In both studies a variety of models was used. A hoki-type model, with average year class strengths, was used as a "baseline"; sensitivity tests included the use of an alternative catch history and non-average year class strengths (in the operating model).

In the estimator performance study, two proportional-squared-error MIAEL estimators of virgin biomass were compared on the basis of their information indices (a measure of their average performance as estimators). The two estimators were the "standard" MIAEL estimator (based on a least squares point estimator), and a "double" MIAEL estimator (identical in structure to the standard estimator, but with the observations first transformed into MIAEL estimates of relative biomass). The results of the study suggest that there may be little to choose between the standard and double MIAEL estimators in estimation performance. However, only a few operating models were tested, so it may be that there are circumstances under which one estimator could substantially outperform the other.

In the resource survey frequency study, six survey patterns were considered starting from a "baseline" assessment in 1997, with later assessments in 2003. It was assumed for 1997 that a relative abundance time series already existed (1992 to 1996 inclusive). Extensions to the time series were considered using different frequencies of relative abundance indices, with the inclusion or not of some absolute abundance indices.

When the assumed estimation model was identical to the operating model, increased survey frequency or the inclusion of absolute abundance indices generally improved the performance of estimators. However, when the estimation and operating models differed, the inclusion of more data or absolute abundance indices sometimes resulted in vastly inferior estimator performance. This lack of robustness, specifically to errors in assumed year class strengths, suggests that it is unlikely that any useful rules of thumb for determining appropriate survey frequencies could be derived for MIAEL estimators

which assume known year class strengths (unless robustness considerations are ignored). Whenever possible, planning of future survey work should be based on individual analyses of each relevant stock (including testing the robustness of the results to estimation model errors).

2. Introduction

This document describes two simulation studies undertaken as part of the stock assessment methods research project in the 1996–97 fishing year (FMFM01). The first study concerns the performance of two estimators of virgin biomass and was done under Objective 1: To evaluate the performance of alternative estimators. The second study concerns the appropriate frequency for relative or absolute biomass surveys and was done under Objective 2: To develop the use of simulation models for resource survey planning.

In Objective 1 it was proposed to investigate alternative stock reduction estimators of virgin biomass and maximum sustainable yield for stocks where relative abundance indices are available from trawl surveys. The aim was to enable optimal estimators to be used in the assessment of many middle depth and inshore stocks for which trawl survey data are currently being collected.

It is easy to develop an estimator for any particular item of interest (a model parameter or a fishery performance index). Such an estimator may be intuitively appealing and/or be based on common statistical practices. However, the most important aspect of an estimator is whether it typically produces "good" estimates (for a given definition of "good"). For instance, a common measure of an estimator's performance is its mean squared error (MSE); one estimator is better than another if it has lower MSE. The generalisation of MSE is expected (estimation) loss for a given loss function (Berger 1985). Normally, amongst competing estimators, no one estimator will be uniformly best over the domain of true values. This problem, combined with a desire to rank estimators, has lead to the development of optimal estimators which minimise in some sense "average" expected loss (where the averaging is done over the range of true values). Optimal Bayes estimators are the best known example, and more recently MIAEL estimators have been developed in a fisheries stock assessment setting (Cordue 1993, 1995, 1998).

MIAEL estimation was developed in a stock reduction setting for hoki stock assessment, and has also been used to assess ling, hake, and gemfish stocks. The principle of obtaining a "best" estimator has been applied only from the stage of given relative biomass indices. For example, trawl survey data are analysed to produce a relative abundance time series, and the MIAEL principle is applied only to the given abundance time series, not the original trawl survey data. It was hoped that the earlier application of MIAEL principles would give estimation benefits that flowed through to the final estimation of virgin biomass, or maximum sustainable yield.

The estimator performance study compares the information indices of two MIAEL estimators for a variety of operating models (a base case model, and a number of variations on it as sensitivity tests). The two estimators are the "standard" MIAEL estimator (based on a least squares point estimator) and a "double" MIAEL estimator (identical in structure to the standard estimator, but with the observations first transformed into MIAEL estimates of relative biomass).

Objective 2 planned to build on earlier modelling work to develop some good rules of thumb for the planning of research surveys. A comparison of the information value of relative abundance indices and absolute abundance indices was planned. The precision and frequency of surveys necessary to obtain "adequate" estimates of virgin and current biomass was to be examined for both absolute and relative abundance indices.

Relative abundance indices, obtained from research surveys or catch per unit effort analysis, are a primary data source for many fisheries stock assessments. Numerous population-dynamics models and estimation procedures have been developed to use relative abundance indices in quantitative stock assessments. However, little work has been done on the research planning aspects of the collection of abundance indices. This is an important issue as fisheries abundance surveys using research vessels are typically expensive.

A formal method for evaluating research survey plans has been developed in the context of MIAEL estimation where an "information index" is calculated for each research survey scenario. This method has been applied to hoki research planning (Cordue 1996). The results clearly demonstrated the additional benefits obtained from absolute rather than relative abundance indices. Generalisations from this work were sought in the hope that some general rules of thumb could be developed regarding the appropriate frequency and precision of surveys

The study of resource survey frequency mainly uses the standard MIAEL estimator to compare information indices for a variety of resource survey scenarios over several operating models (a maximum likelihood variation was also briefly investigated). In this study, estimation of both current and virgin biomass is considered.

3. Methods

This section describes the population model used in the work (*see* Appendix 1 for the equations), and details the scope and methods of the two simulation studies. Both studies use similar methods, including the same population model. The MIAEL estimators used in the frequency investigation are based on two types of point estimators: a least squares and a maximum likelihood estimator (*see* Appendix 2 for details). The "double" MIAEL estimator, which is compared with the standard MIAEL estimator in the performance study, is derived by transforming the observations to MIAEL estimates of relative abundance in the sum of squares (Appendix 3) and then applying MIAEL estimation to the transformed observations. All of the MIAEL estimators used in the studies are *best p* estimators using a proportional-squared-error loss function (*see* Appendix 4 for *best p* estimators and the formulation of the information index).

3.1 Model hierarchy

It is useful to bear in mind the hierarchy of "models" which is involved in these simulation studies. At the highest level is a population model structure which defines a framework within which particular models are created. The framework sets out the relationships between the parameters of the model—particular parameter values are not specified, just the type of parameters and their relationship to each other.

At the next level are fully or partially specified models where particular values are specified for all or most of the parameters. For example, from a given framework an orange roughy population model could be defined by specifying a suitable maximum age and appropriate growth parameters; similarly, with different parameter values, a hoki model or a snapper model could be defined.

Whenever population models are used for estimation purposes, then in addition to the population model there is a "data model". This includes the available observations and a precise description of how they relate to the (model) population. For example, a time series of acoustic surveys might be taken to be absolute indices of spawning biomass, with each year's index assumed to be normally distributed about the true spawning biomass.

The terms "operating model" and "estimation model" are used in this document. For the purposes of simulation, involving estimation of population parameters, an "operating model" is a partially specified population model *and* a corresponding data model. When values are specified for the unknown parameters of an operating model, simulated data can be generated using the data model. By simulating over a range of true parameter values, one or more estimators can be evaluated for average performance or their estimation properties. For example, the bias of an estimator is approximated for a particular realisation of an operating model by performing a large number of simulations using the given data model, and on each simulation run calculating the estimate and then subtracting the average estimate from the known value of whatever is being estimated. If this is done for various true parameter values then an average bias can be calculated.

An "estimation model", in this document, refers to a partially specified population model and a corresponding data model, which are used for the purposes of estimation. For an initial point estimator (on which a MIAEL estimator is based) the unknown parameters in the model (i.e., those not specified) are estimated by searching for parameter values which generate predicted values (i.e., from the model) which in some sense best "fit" the available observations (from the data model).

In the studies in this document, the operating models and estimation models all have a common population model structure (described in Section 3.2) with virgin biomass unspecified. The estimation and operating population models can therefore differ only in the values that the specified parameters are assigned. The only deviations in parameter values considered in this document are those of year class strengths. In the estimation models the year class strengths are always equal to unity, but in some operating models non average year class strengths are used.

3.2 Population model structure

The population model is a spatially disaggregated adaptation of a succession of deterministic two-sex age-structured single-stock models (Cordue *et al.* 1992, Cordue 1993, 1994, 1995). The earlier models were successive refinements of that of Mace & Doonan (1988). This model has a multi-stage annual cycle incorporating a two-stage pre-spawning season and a two-stage spawning season. The length of the spawning season is specified and the spawning and pre-spawning seasons are each split into halves. Three areas are defined: a spawning ground; a home ground, which is also the nursery ground; and a "corridor", which is used to move fish between the spawning ground and the nursery. The stock is assumed to have a Beverton-Holt spawning-biomass to recruitment relationship. Fishing occurs in the spawning ground during the spawning season and in the home ground during the pre-spawning season. There is no fishing in the corridor.

3.2.1 Annual cycle

In the virgin state the fish population is in deterministic equilibrium: fish move between areas during their annual cycle, but recruitment to the stock results in exactly the same number of fish in each area for any given point in the annual cycle. When fishing is introduced into the system, the annual cycle remains the same, but the equilibrium is interrupted.

The cycle is described below. Four terms are used to denote members of the fish population: "larvae", "juveniles", "adults", and "fish". "Larvae" are spawned on the spawning ground; they are the new entrants to the population. When they are 1 year old they become "juveniles". At some stage "juveniles" mature and become "adults". The term "fish" denotes "juveniles" or "adults" (not "larvae").

Stage 1: *Beginning of year*

- Fishing year begins.

Stage 2: *Corridor migrations and maturation*

- Larvae move from the spawning ground to the corridor.
- Some juveniles move from the corridor to the nursery.
- Some juveniles mature (and become adults).

Stage 3: *Pre-spawning season: first half*

- Natural and fishing mortality are applied to fish in the home ground.

- Natural mortality is applied to fish in the corridor.

Stage 4: *Pre-spawning season: second half*

- Natural and fishing mortality are applied to fish in the home ground.
- Natural mortality is applied to fish in the corridor.

Stage 5: *Ageing and spawning migration*

- Fish become 1 year older.
- Larvae become 1 year old juveniles
- Some adults move from the home ground to the spawning ground.

Stage 6: *Spawning season: first half*

- Natural and fishing mortality are applied to fish in the spawning ground.
- Natural mortality is applied to fish in the corridor and the home ground.
- Larvae are created in the spawning ground (based on the mature female biomass present).
- The sex of larvae is determined.

Stage 7: *Spawning season: second half*

- Natural and fishing mortality are applied to fish in the spawning ground.
- Natural mortality is applied to fish in the corridor and the home ground.

Stage 8: *End of year*

- Adults return from the spawning ground to their home ground.

3.2.2 Model parameters

In the following the subscript i always refers to the ith fishing year. Ages are subscripted by j and sexes are subscripted by s.

Size of the fish stocks

B_0	Equilibrium mid spawning-season biomass in the spawning ground.
R_i	The recruitment strength multipliers: multipliers of the total number of larvae as given by the Beverton-Holt stock recruitment relationship.

Cycle parameters

Stage 1:

None.

Stage 2:

$nurs_j$	The proportions of juveniles which move from the corridor to the nursery.
$m\_og_{s,j}$	The proportions of immature fish which become mature.

Stage 3:

sp_length	The length of the spawning season as a proportion of the year; thus the length of each half of the pre-spawning season is $0.5 * (1 - sp\_length)$.
$u_{s,j}$	The mean weights of fish during the pre-spawning season.
M_s	The instantaneous rates of natural mortality.
max_hm	The maximum exploitation rate (ratio of catch to beginning of pre-spawning season biomass) for the home ground.
$sel\_hm_{s,j}$	The relative fishing selectivities for fish in the home ground.

Stage 4:

See stage 3.

Stage 5:

$spawn\_p$	The proportion of mature fish which migrate to the spawning ground.

Stage 6:

sp_length	The length of the spawning season as a proportion of the year; the length of each half of the spawning season is 0.5 * sp_length.
$w_{s,j}$	The mean weights of fish during the spawning season.
M_s	The instantaneous rates of natural mortality.
max_sp	The maximum exploitation rate (ratio of catch to beginning of spawning season biomass) for the spawning ground, assuming that all spawning fish are available to the fleet. (The assumption allows for the possibility that part of the spawning ground could be closed to fishing.)
p_out_i	The average proportions of spawning biomass available to the fleet during the spawning season.
$sel\_sp_{s,j}$	The relative fishing selectivities for fish in the spawning ground.
$steep$	The steepness of the Beverton-Holt stock recruit relationship (based on female biomass in the spawning ground).
p_male	The proportion of larvae that are male.

Stage 7:

See stage 6.

Stage 8:

No specific parameters: all of the spawning fish return to their home grounds.

3.2.3 Fishing mortality

During the pre-spawning season fishing can occur only in the home ground. During the spawning season fishing occurs only in the spawning ground. No fishing occurs in the corridor. For each of the grounds in which fishing can occur, there is a sex and age specific selection ogive (Section 3.2.2) which is always proportional to the instantaneous fishing mortalities applied to each sex specific age class. When both adults and juveniles are present on a ground during fishing, the fishing mortality is applied equally across the maturity categories (i.e., fishing selection is not a function of maturity). Limits on the exploitation rate are imposed for the home ground (Section 3.2.2, Stage 3). Similarly, there is a limit on the spawning ground exploitation rate (Section 3.2.2, Stage 6), though there is a complication because of an allowance for a possible closed area (see Cordue *et al.* 1992).

3.2.4 Calculation of fishing mortality

For a given fishing period (pre-spawning or spawning season in some year) constant age and sex specific instantaneous fishing mortalities ($F_{s,j}$ for sex s and age j) are assumed. This results in the familiar Baranov catch equation. Given a catch by weight of $W(t)$ over a time period t the fishing mortalities $F_{s,j} = F\ s_{s,j}$ are calculated from the known selectivities $s_{s,j}$ by iteratively solving the following equation for F:

$$F = \frac{W(t)}{\sum_{s,j} \frac{s_{s,j}}{(M_s + s_{s,j} F)} N_{s,j} \left[1 - \mathrm{e}^{-t(M_s + s_{s,j} F)}\right] a_{s,j}}$$

where the M_s are the natural mortalities, the $a_{s,j}$ are the average fish weights, and the $N_{s,j}$ are the numbers of fish at the start of the period. Note that $W(t)$ is limited by the restriction on the catch to beginning-of-period biomass ratio. When the specified catch is too large, the maximum possible catch is substituted.

3.2.5 Equilibrium distribution

In the virgin state the stock is assumed to be in deterministic equilibrium: each year, for any given point in the annual cycle, there are the same number of fish in every category (i.e., by sex, age, maturity, and area). Explicit formulae giving the number of fish in each category at each time point in the cycle can be complex and have not been derived. The key technical point of interest is that the equilibrium mid spawning-season biomass (B_0) can be expressed in terms of the total number of larvae produced in the spawning ground by the equation:

$$B_0 = aN$$

where N is the total number of larvae and a is the spawning biomass per recruit (a can be derived by calculating the equilibrium spawning biomass resulting from $N = 1$). The constant a depends on the cycle parameters (Section 3.2.2). Hence, for any given equilibrium biomass, the above equation can be solved for "virgin recruitment" (N) and the equilibrium distribution can be put in place.

3.3 The baseline model

In both studies, “hoki” type baseline operating and estimation population models were used. For the most part the operating and estimation models were the same. The population model parameters were chosen so that the model was similar to that used in recent assessments of the western hoki stock (Ballara *et al.* 1997, 1998), except that average year class strengths were used and the closed area covering part of the spawning ground was ignored. There are minor differences in the biological parameters due to the somewhat different requirements of the new model and other minor adjustments (Table 1). Extra biological parameters needed by the new model are the spawning season length, the proportion of male larvae at birth, and the corridor migration ogive. The other biological parameters are common to both the old and new models and were given the same values, except that the proportion spawning was rounded up from 77% to 80%, and a slightly flatter maturity ogive was used (based on estimates obtained using a multi-stock hoki model — author's unpublished results).

The fishery parameters of the new model are the same as in the old model, except that an explicit selectivity for the home ground is defined (Table 2). In the previous model, maturity was assumed to coincide with recruitment. This ignored the fishing on juveniles in the home ground (i.e., the Chatham Rise) during the pre-spawning season. The selectivity in the new model explicitly allows for this, but makes the younger fish far less vulnerable than the older fish (*see* Table 2, the vulnerability of hoki is assumed to peak at ages 5 and 6 — based on the author's unpublished estimates from a multi-stock hoki model).

The annual parameters of the baseline population model differ from those used for the western hoki stock: the catch history is the same (Table 3) but average year class strengths and full availability of the spawning fish to the fleet are assumed (i.e., no closed area on the spawning ground).

Three alternative scenarios were also used in the studies: an alternative catch history; generally below average year class strengths; and generally above average year class strengths (Table 3). The non-average year-class-strengths were used only in operating models (with the baseline model used as the estimation model in those cases). The alternative catch history was used simultaneously in associated operating and estimation models.

3.4 Data models and parameter estimation

In both studies, only two parameters were estimated, virgin mid-spawning season biomass (B_0) and the proportionality constant for the relative biomass index (q). For the purposes of simulation and estimation, bounds were placed on B_0 in the operating and estimation models, respectively (Table 4). The alternative catch history model has a much lower range on B_0 than the baseline model, it models a much smaller stock. The lower bound on B_0 of the non-average year-class-strength models differs from that of the baseline model. The lower bounds in the operating models are approximately equal to the minimum levels of virgin biomass which would allow the full catch histories to be removed, given the maximum catch to biomass ratios assumed (*see* Table 2). Hence, with lower than average year class strengths, a higher minimum biomass results, and with higher than average year class strengths, there is a lower minimum biomass (Table 4).

In both studies a variety of research survey scenarios are considered, six in total. Two assessment years are considered, 1997 and 2003. For 1997 the available observations are a single relative abundance time series from 1992 to 1996 inclusive. For the assessments in 2003, five further scenarios are considered: an extension of the relative abundance time series, either annually, biennially, or triennially, and triennial surveys with either one or two absolute abundance estimates (Table 5).

Each index in the relative abundance time series was assumed to be normally distributed with a *c.v.* of 25%. The absolute abundance indices were assumed to be normally distributed with a *c.v.* of 35%. In both cases the indices were assumed to relate to mid-spawning season biomass.

The results of each study consist of a collection of information indices calculated for a variety of combinations of research survey scenarios and alternative MIAEL estimators. The research survey scenarios, or "index patterns", have already been described. The estimators considered were: the "standard" MIAEL estimator (based on a least squares estimator, with equal source weights for each time series; *see* Appendix 2); two "double" MIAEL estimators (based on MIAEL estimates of the observations, with two different assumptions about the bounds on q; *see* Appendix 3); and a MIAEL estimator based on the maximum likelihood estimator (*see* Appendix 2). All the MIAEL estimators used a proportional-squared-error loss function and a *best p* formulation (*see* Appendix 4). In the estimator comparison study only estimation of virgin biomass was considered. In the other study, both virgin biomass (B_0) and current biomass ($B_{current}$) were done.

The maximum likelihood MIAEL estimator was used only four times: with the baseline model for estimation of B_0 and $B_{current}$ for both the 1997 assessment year and the annual extension in the 2003 assessment year. Since, in all four cases, the information indices were virtually identical to those of the standard MIAEL estimator, investigation of the maximum likelihood MIAEL estimator was not continued.

In the estimator comparison study, two "double" MIAEL estimators were initially used, each based on a different assumption about the bounds on q: (a) $q \varepsilon [.1, 2]$ and (b) $q \varepsilon [.5, 10]$. (Note, in the simulations the true value of q is always 1.) The two estimators gave very similar information indices for B_0 when the baseline model was used with the relative abundance time series scenarios. The second double MIAEL estimator was used exclusively in the later comparisons (with the standard MIAEL estimator).

Stratified random sampling was used to obtain an approximate information index for each MIAEL estimator of B_0 (*see* Appendix 4 for the basic formulation of an information index). This method allows estimation of the precision obtained on the information index, and this was done for some of the information indices to ensure that adequate precision was obtained for the studies. The bounds on B_0 in the operating model were split into six equal intervals. Random values of B_0 were generated within the bounds, using a uniform distribution, until there were exactly two values within each of the intervals (the same seed was used for all models in the generation of B_0s, so that models with the same bounds used the same B_0s). At each of the 12 values of B_0, 100 simulated point estimates were generated (using the given estimation model and the point estimator associated with the given MIAEL estimator. The simulated data were creating using different seeds for each B_0, but the same seeds were used across all models). Where the operating and estimation models were the same, the approximate information index of the *best p* estimator was calculated directly from the simulated estimates. The *best p* estimator was determined by searching for the value of p which minimised, within the *best p* class of candidate estimators, the average proportional mean squared error (i.e., averaged over the 12 points; this approximates the Integrated Average Expected Loss for a proportional squared error loss function—*see* Cordue 1995).

When the estimation model differed from the operating model, two information indices were determined: that which would be "reported" if the operating model were assumed to be identical to the estimation model (obtained as above, from the *best p* estimator when the models are the same); and the "actual" information index. The actual information index was calculated (using the correct operating model) as the information index of the *best p* estimator which gave rise to the reported information index, that is, using the value of p which is optimal for the estimation model when used as the operating model, rather than using the optimal value of p for the actual operating model.

The calculation of information indices for MIAEL estimators of $B_{current}$ was analogous to that for B_0 (the process is the same, but all B_0s must be mapped to $B_{current}$s, be they estimates or true values). Note, $B_{current}$ is a function of B_0, within both the operating model (governing true values) and the estimation model (governing estimates). The main difference in calculating information indices for $B_{current}$ is in the generation of the random values of B_0. Bounds on $B_{current}$ were determined from the bounds on B_0 (in the operating model) and these bounds were divided into six equal intervals. Random values of B_0 were generated using a uniform distribution until there were exactly two corresponding values of $B_{current}$ in each interval. One hundred estimates of B_0 were generated at each of the 12 points, and these were transformed into estimates of $B_{current}$. The *best p* estimators and information indices were then determined as described above for B_0. (Note, $B_{current}$ is an approximately linear function of B_0 (Cordue, unpublished results), so the selected values of $B_{current}$ are approximately uniformly distributed.)

4. Results and discussion

4.1 Estimator performance comparison

Although only approximate information indices were calculated, strict comparisons between the information indices of different estimators can be made because in each case the same B_0s have been used with the same simulated data (on each of the 100 runs at each B_0).

The information indices for the standard MIAEL estimator and the two double MIAEL estimators for B_0, using the baseline model, are very similar between estimators for each of the relative abundance index patterns (Table 6). The double MIAEL estimators are always as good as, or slightly better than, the standard MIAEL estimator. For the same scenarios under the alternative catch history model, the double MIAEL estimator (with $q \varepsilon [.5, 10]$) is also slightly better than the standard MIAEL estimator (Table 7).

When the operating model differs from the estimation model, under either non-average year class strength assumption, the double MIAEL estimator performs the same as, or slightly worse than, the standard MIAEL estimator (Tables 8 and 9).

Thus, in comparative terms the double MIAEL estimator was slightly better than the standard MIAEL estimator when the estimation model was "correct" and slightly worse when the estimation model was "incorrect". It appears that any differences in estimation performance are likely to be small. However, only a few operating models were tested, so it may be that there are circumstances under which one estimator could substantially outperform the other.

4.2 Resource survey frequency study

For this study, comparisons of the information indices of the standard MIAEL estimators of B_0 and $B_{current}$ were made between different index patterns (*see* Table 5). The questions are: how much information is lost when surveys are conducted triennially, or biennially, instead of annually?; how much more information is gained when absolute abundance indices are obtained instead of relative abundance indices?; and how robust are the answers to these questions if the estimation models are incorrect?

The information indices calculated to answer these questions are only approximate and their precision must be taken into account when interpreting the results. The precision of the calculated information indices decreases with magnitude. Smaller information indices correspond to expected loss functions which vary greatly with B_0 and so have higher "spatial variability" than larger information indices. To get an idea of the precision of the indices, approximate radius were calculated for 95% confidence intervals on the information indices for the research scenarios on the baseline model. These varied from 1 to 10% (e.g., an information index of 9% was ± 7%, and an information index of 76% was ● 2%).

Under the baseline model, the information indices for the standard MIAEL estimator of B_0 increase steadily as increased numbers of relative abundance surveys are used in the index pattern (*see* Table 6). This pattern of increase is plausible, but as the information indices are all very low, their precision is not adequate to infer any real trend. Under the alternative catch history model, triennial, biennial, or annual surveys all perform similarly and provide definite improvement over the 1997 assessment (*see* Table 7). When non-average year class strengths are used in the operating model (with the baseline catch history) all the information indices are near to zero (*see* Tables 8 & 9) and the actual information indices are somewhat lower than the reported information indices (Table 10).

For estimation of $B_{current}$, annual surveys are probably best under the baseline model (but all the information indices are very low) and are definitely best for the alternative catch history (Table 11). Also, for the alternative catch history, triennial surveys are as good as biennial surveys with both giving a large improvement over the 1997 assessment (Table 11).

When estimating $B_{current}$ with the operating model using the lower than average year class strengths, there are only minor differences between the reported and actual information indices (Table 12). However, when the above average year class strengths are used, there are very large differences between the reported and actual information indices, with the actual information indices all negative, and becoming more so with an increasing number of surveys (Table 12). This is a good demonstration of two points that are intuitively obvious. First, with only relative abundance indices available and no data on year class strengths, current biomass is generally more difficult to estimate than virgin biomass. Second, incorporating more data into an estimation procedure does not guarantee improved estimation performance; an incorrect interpretation of data (because of faulty assumptions) can mean that the larger the data set, the worse the error.

For both the baseline and alternative catch history models, when absolute biomass indices are available the information indices for the standard MIAEL estimators of B_0 and $B_{current}$ are much higher than those achieved using only relative abundance indices (Tables 13 and 14). For B_0, two absolute indices gave slightly higher information indices than only one absolute index (Table 13). For $B_{current}$, the information indices were very similar, independent of the number of absolute indices (Table 14).

Under the non-average year class strength operating models, when estimating B_0, the actual information indices are still fairly high in comparison to those obtained using only relative abundance indices (reported or actual), but they are much lower than the reported information indices (Table 15). Also, the estimator performs better with the higher, rather than the lower, than average year class strengths. The same cannot be said when estimating $B_{current}$ (Table 16). Under the lower than average year class strengths the information indices are much lower than those reported, but they are still positive. However, for the higher than average year class strengths, the information indices are spectacularly negative, with the worst information indices corresponding to the patterns which include one or two absolute indices (Table 16).

The results for the above average year class strengths are due to a combination of factors. The higher than average year class strengths mean that the minimum bound on B_0 is lower in the operating model than the estimation model (*see* Table 4). This in turn leads to a lower minimum bound on $B_{current}$. Hence MIAEL estimators will tend to overestimate both B_0 and $B_{current}$. Because a proportional squared error loss function is used, a consistent positive bias is more damaging than a consistent negative bias. The large negative information indices obtained when the absolute indices are included are somewhat surprising. Intuitively one would expect not too bad a result with an absolute abundance index in the current year. However, the weightings given to individual indices in the sum of squares is obviously important, particularly when the wrong year class strengths are being used in the estimation model. That is, when the year class strengths are wrong, better performance will probably be achieved in estimating current biomass by giving more weight to recent or current absolute abundance indices, rather than any relative abundance indices (whose correct interpretation depends on accurate knowledge of the year class strengths).

5. Conclusions

The results of the estimator performance study suggest that there may be little to choose between the standard and double MIAEL estimators. However, only a few operating models were tested, so it may be that there are circumstances under which one estimator could substantially outperform the other.

When the assumed estimation model was identical to the operating model it generally followed that increased survey frequency or the inclusion of absolute abundance indices improved the performance of estimators. However, when the estimation and operating models differed, the inclusion of more data, or absolute abundance indices, sometimes resulted in vastly inferior estimator performance. This lack of robustness, specifically to errors in assumed year class strengths, suggest that it is unlikely that any useful rules of thumb for determining appropriate survey frequencies could be derived for MIAEL estimators which assume known year class strengths (unless robustness considerations are ignored). Whenever possible, planning of future survey work should be based on individual analyses of each relevant stock (including testing the robustness of the results to estimation model errors).

There are some other lessons to be learnt from this work. It appears that the estimation of virgin biomass is quite robust to year class strength errors in the estimation model, although the level of certainty is likely to be over-stated. However, the estimation of current biomass can be very sensitive to year class strength errors. When both absolute and relative abundance indices are available for estimation of current biomass, it appears that the best results may be obtained by placing most weight on the most recent absolute abundance index.

6. Acknowledgments

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Table 1: Baseline model parameters: biological

Maximum age:	20 years
Steepness:	0.9
Spawning season:	3 months
Proportion of male larvae:	0.5
Proportion spawning:	0.8

Corridor migration:

Age:	1	2	3
Proportion that move:	0.6	0.9	1.0

Maturity (proportion that mature):

Age:	2	3	4	5	6
Male:	0.1	0.5	0.8	1.0	1.0
Female:	0.05	0.1	0.5	0.8	1.0

Growth, natural mortality, and length-weight parameters:

	M (yr^{-1})	L_∞ (cm)	K (yr^{-1})	t_0 (yr)	a	b
Male:	0.30	92.6	0.261	-0.5	0.006	2.85
Female:	0.25	104.0	0.213	-0.6	0.006	2.85

Table 2: Baseline model parameters: fishery

Home ground selectivity:

Age:	1	2	3	4	5	6	7	8^+
Male:	0.1	0.2	0.5	0.8	1.0	1.0	0.9	0.8
Female:	0.1	0.2	0.5	0.8	1.0	1.0	0.9	0.8

Spawning ground selectivity:

Age:	3	4	5	6	7	8^+
Male:	1.0	1.0	1.0	1.0	1.0	1.0
Female:	1.0	1.0	1.0	1.0	1.0	1.0

Maximum catch to biomass ratios:

Pre-spawning season:	0.6
Spawning season:	0.8

Table 3: The baseline and alternative catch histories, and alternative year class strengths (YCS) used in operating models when they differed from the estimation model. In the estimation models year class strengths were all equal to 1

	Baseline catch history		Alternative catch history			
Year	Pre-spawning catch ('000 t)	Spawning catch ('000 t)	Pre-spawning catch ('000 t)	Spawning catch ('000 t)	YCS (a)	YCS (b)
1972	0	5	0	5	1	1
1973	0	5	0	5	1	1
1974	0	5	0	5	1	1
1975	0	10	0	10	1	1
1976	0	30	0	30	1	1
1977	0	60	0	60	1	1
1978	0	5	0	5	1	1
1979	0	18	0	18	1	1
1980	0	20	0	20	1	1
1981	0	25	0	25	1	1
1982	0	25	0	25	0.8	1.8
1983	0	30	0	30	0.8	1.8
1984	0	40	0	40	0.5	1.2
1985	0	34	0	34	0.5	1.2
1986	0	82	0	50	1.2	0.8
1987	0	158	0	50	1.2	0.8
1988	0	240	0	50	0.8	1.8
1989	0	192	0	50	0.8	1.8
1990	8	175	8	50	0.5	1.2
1991	12	164	12	50	0.5	1.2
1992	27	112	15	50	1.2	0.8
1993	24	102	15	50	1.2	0.8
1994	15	114	15	50	0.8	1.8
1995	13	81	15	50	0.8	1.8
1996	12	78	15	50	0.5	1.2
1997	25	100	15	50	0.5	1.2
1998	25	100	15	50	1.2	0.8
1999	25	100	15	50	1.2	0.8
2000	25	100	15	50	0.8	1.8
2001	25	100	15	50	0.8	1.8
2002	25	100	15	50	1	1
2003	25	100	15	50	1	1

Table 4: Lower and upper bounds on virgin biomass (B_0) used in the operating and estimation models

	Operating model		Estimation model	
	Lower bound ('000 000 t)	Upper bound ('000 000 t)	Lower bound ('000 000 t)	Upper bound ('000 000 t)
Baseline	0.7	2.0	0.7	2.0
Alternative catch history	0.36	1.2	0.36	1.2
Lower YCS	0.9	2.0	0.7	2.0
Higher YCS	0.6	2.0	0.7	2.0

Table 5: The index patterns considered. The years in which observations are obtained is given for each pattern. Years appearing in bold characters denote absolute indices, other indices are relative. The relative indices are assumed to continue the existing times series (1992 to 1996 inclusive)

Index years (beyond 1996):

Annual:	1997	1998	1999	2000	2001	2002
Biennial:	-	1998	-	2000	-	2002
Triennial:	-	-	1999	-	-	2002
Absolute 1:	-	-	1999	-	-	**2002**
Absolute 2:	-	-	**1999**	-	-	**2002**

Table 6: Baseline model: comparison of information indices for MIAEL estimators of B_0 for various index patterns. Indices are given for the standard MIAEL estimator and the "double" MIAEL estimator for two different assumptions about the proportionality constant q: (a) q ε [.1, 2], (b) q ε [.5, 10]

Assessment year	Index pattern (beyond 1996)	MIAEL information index (%)	Double MIAEL (a) information index (%)	Double MIAEL (b) information index (%)
1997	-	1	2	2
2003	triennial	5	5	6
2003	biennial	9	9	10
2003	annual	14	15	16

Table 7: Alternative catch history: comparison of information indices for MIAEL estimators of B_0 for various index patterns. Indices are given for the standard MIAEL estimator and the "double" MIAEL estimator with the assumption that the proportionality constant q ε [.5, 10]

Assessment year	Index pattern (beyond 1996)	MIAEL information index (%)	Double MIAEL information index (%)
1997	-	5	7
2003	Triennial	26	29
2003	Biennial	26	29
2003	Annual	30	32

Table 8: **Lower than average year class strengths in the operating model: comparison of information indices for MIAEL estimators of B_0 for various index patterns. Indices are given for the standard MIAEL estimator and the "double" MIAEL estimator with the assumption that the proportionality constant q ε [.5, 10]**

Assessment year	Index pattern (beyond 1996)	MIAEL information index (%)	Double MIAEL information index (%)
1997	-	-4	-5
2003	triennial	-2	-3
2003	biennial	0	0
2003	annual	3	2

Table 9: **Higher than average year class strengths in the operating model: comparison of information indices for MIAEL estimators of B_0 for various index patterns. Indices are given for the standard MIAEL estimator and the "double" MIAEL estimator with the assumption that the proportionality constant q ε [.5, 10]**

Assessment year	Index pattern (beyond 1996)	MIAEL information index (%)	Double MIAEL information index (%)
1997	-	2	2
2003	triennial	-1	-2
2003	biennial	1	0
2003	annual	-5	-7

Table 10: **Comparison of actual and reported information indices for the standard MIAEL estimator of B_0 for various index patterns. The actual indices are for operating models that differed from the estimation model: (a) lower than average year class strengths in the operating model; (b) higher than average year class strengths in the operating model**

Assessment year	Index pattern (beyond 1996)	Reported MIAEL information index (%)	Actual MIAEL information index (%) (a)	Actual MIAEL information index (%) (b)
1997	-	1	-4	2
2003	triennial	5	-2	-1
2003	biennial	9	0	1
2003	annual	14	3	-5

Table 11: **Comparison of information indices for the standard MIAEL estimator of $B_{current}$ for various index patterns. Indices are given for (a) the baseline model and (b) the alternative catch history**

Assessment year	Index pattern (beyond 1996)	MIAEL information index (%) (a)	MIAEL information index (%) (b)
1997	-	4	3
2003	Triennial	5	31
2003	biennial	9	28
2003	annual	12	40

Table 12: **Comparison of actual and reported information indices for the standard MIAEL estimator of $B_{current}$ for various index patterns. The actual indices are for operating models that differed from the estimation model: (a) lower than average year class strengths in the operating model; (b) higher than average year class strengths in the operating model**

Assessment year	Index pattern (beyond 1996)	Reported MIAEL information index (%)	Actual MIAEL information index (%) (a)	Actual MIAEL information index (%) (b)
1997	-	4	-6	-43
2003	triennial	5	2	-88
2003	biennial	9	5	-154
2003	annual	12	9	-253

Table 13: **Comparison of information indices for the standard MIAEL estimator of B_0 for various index patterns. Indices are given for (a) the baseline model and (b) the alternative catch history**

Assessment year	Index pattern (beyond 1996)	MIAEL information index (%) (a)	MIAEL information index (%) (b)
2003	Annual	14	30
2003	Triennial	5	26
2003	Absolute 1	76	77
2003	Absolute 2	82	83

Table 14: **Comparison of information indices for the standard MIAEL estimator of $B_{current}$ for various index patterns. Indices are given for (a) the baseline model and (b) the alternative catch history**

Assessment year	Index pattern (beyond 1996)	MIAEL information index (%) (a)	MIAEL information index (%) (b)
2003	annual	12	40
2003	triennial	5	31
2003	absolute 1	83	82
2003	absolute 2	82	86

Table 15: **Comparison of actual and reported information indices for the standard MIAEL estimator of B_0 for various index patterns. The actual indices are for operating models that differed from the estimation model: (a) lower than average year class strengths in the operating model; (b) higher than average year class strengths in the operating model**

Assessment year	Index pattern (beyond 1996)	Reported MIAEL information index (%)	Actual MIAEL information index (%) (a)	Actual MIAEL information index (%) (b)
2003	annual	14	3	-5
2003	triennial	5	-2	-1
2003	absolute 1	76	25	47
2003	absolute 2	82	28	53

Table 16: **Comparison of actual and reported information indices for the standard MIAEL estimator of $B_{current}$ for various index patterns. The actual indices are for operating models that differed from the estimation model: (a) lower than average year class strengths in the operating model; (b) higher than average year class strengths in the operating model**

Assessment year	Index pattern (beyond 1996)	Reported MIAEL information index (%)	Actual MIAEL information index (%) (a)	Actual MIAEL information index (%) (b)
2003	annual	12	9	-253
2003	triennial	5	2	-88
2003	absolute 1	83	20	-514
2003	absolute 2	82	29	-410

Appendix 1: The single stock population model

In the population dynamics model, fish are categorised by ground, sex, age, and maturity. Given the level of complexity of the categorisation it is best to present the mathematical equations in a very descriptive form using categorical variables. Categorical variables are given in italics and specific members of a category (except ages) are given in bold italics.

The following abbreviations are used in the category member names:

"spawning" = ***sp***, "home" = ***hm***, "corridor" = ***cor***, "female" = ***fem***, "immature" = ***imm***, "mature" = ***mat***, "maximum age" = ***amax***.

The categorical variables and their associated categories are:

ground	{ ***sp***, ***hm***, ***cor*** }
sex	{ ***male***, ***fem*** }
age	{ 0, 1, ... , ***amax*** }
maturity	{ ***imm***, ***mat*** }

The numbers of fish in each category in year *i* and cycle point *j* are denoted by $N_{i,j}(\mathit{ground}, \mathit{sex}, \mathit{age}, \mathit{maturity})$. Unless otherwise stated, an equation involving one or more categorical variables is valid for each member of the associated category or categories (where the particular combination of values is valid; an example of an invalid combination is "mature fish aged 0 years"). Equations are applied consecutively. Note, equations of the form "A += B" are shorthand for "$A = A + B$". Similarly for "A –= B" and "A *= B".

Stage 1: Beginning of fishing year

$$N_{i,1}(\mathit{ground}, \mathit{sex}, \mathit{age}, \mathit{maturity}) = N_{i-1,8}(\mathit{ground}, \mathit{sex}, \mathit{age}, \mathit{maturity})$$

Stage 2: Corridor migrations and maturity

$$N_{i,2}(\mathit{ground}, \mathit{sex}, \mathit{age}, \mathit{maturity}) = N_{i,1}(\mathit{ground}, \mathit{sex}, \mathit{age}, \mathit{maturity})$$

(a) Larvae move from the spawning ground to the corridor.

$$N_{i,2}(\boldsymbol{\mathit{cor}}, \mathit{sex}, 0, \boldsymbol{\mathit{imm}}) = N_{i,2}(\boldsymbol{\mathit{sp}}, \mathit{sex}, 0, \boldsymbol{\mathit{imm}})$$

$$N_{i,2}(\boldsymbol{\mathit{sp}}, \mathit{sex}, 0, \boldsymbol{\mathit{imm}}) = 0$$

(b) Some juveniles move from the corridor to the nursery (home).

$$N_{i,2}(\boldsymbol{\mathit{hm}}, \mathit{sex}, \mathit{age}, \boldsymbol{\mathit{imm}}) \mathrel{+}= \mathit{nurs}(\mathit{age}) * N_{i,2}(\boldsymbol{\mathit{cor}}, \mathit{sex}, \mathit{age}, \boldsymbol{\mathit{imm}})$$

$$N_{i,2}\,(\boldsymbol{cor},\ sex,\ age,\ \boldsymbol{imm}) \mathrel{-}= nurs(age)\ *\ N_{i,2}\,(\boldsymbol{cor},\ sex,\ age,\ \boldsymbol{imm})$$

(c) Some juveniles mature (and become adults).

For $age \geq 1$

$$N_{i,2}\,(\boldsymbol{hm},\ sex,\ age,\ \boldsymbol{mat}) \mathrel{+}= m\_og(\ sex,\ age)\ *\ N_{i,2}\,(\boldsymbol{hm},\ sex,\ age,\ \boldsymbol{imm})$$

$$N_{i,2}\,(\boldsymbol{hm},\ sex,\ age,\ \boldsymbol{imm}) \mathrel{-}= m\_og(\ sex,\ age)\ *\ N_{i,2}\,(\boldsymbol{hm},\ sex,\ age,\ \boldsymbol{imm})$$

Stage 3: Pre-spawning season: first half

$$N_{i,3}\,(ground,\ sex,\ age,\ maturity) = N_{i,2}\,(ground,\ sex,\ age,\ maturity)$$

(a) Natural and fishing mortality are applied to fish in the home ground.

For $age \geq 1$

$$N_{i,3}\,(\boldsymbol{hm},\ sex,\ age,\ maturity) \mathrel{*}= \exp[\ -t\ *\ (\ F(sex,\ age) + M(sex)\ )\ ]$$

where the fishing mortalities $F(sex,\ age)$ are calculated from the Baranov catch equation using the selectivities $sel\_hm(sex,\ age)$ and the pre-spawning season catch. The time period t in this case is $0.5\ *\ (\ 1 - sp\_length\ )$.

(b) Natural mortality is applied to fish in the corridor.

For $age \geq 1$

$$N_{i,3}\,(\boldsymbol{cor},\ sex,\ age,\ maturity) \mathrel{*}= \exp[\ -t\ *\ M(sex)\ ]$$

Stage 4: Pre-spawning season: second half

$$N_{i,4}\,(\ ground,\ sex,\ age,\ maturity) = N_{i,3}\,(\ ground,\ sex,\ age,\ maturity)$$

(a) Natural and fishing mortality are applied to fish in the home ground.

For $age \geq 1$

$$N_{i,4}\,(\boldsymbol{hm},\ sex,\ age,\ maturity) \mathrel{*}= \exp[\ -t\ *\ (\ F(sex,\ age) + M(sex)\ )\ ]$$

where the $F(sex,\ age)$ and t are as in Stage 3.

(b) Natural mortality is applied to fish in the corridor.

For $age \geq 1$

$N_{i,4}\,(\boldsymbol{cor}, sex, age, maturity)$ *= $\exp[\,-t * M(sex)\,]$

Stage 5: Ageing and spawning migration

$N_{i,5}\,(ground, sex, age, maturity) = N_{i,4}\,(ground, sex, age, maturity)$

(a) Fish and larvae age 1 year.

$N_{i,5}\,(ground, sex, amax, maturity)$ += $N_{i,5}\,(ground, sex, amax - 1, maturity)$

For $age = (amax - 1)$ down to $age = 1$

$N_{i,5}\,(ground, sex, age, maturity) = N_{i,5}\,(ground, sex, age - 1, maturity)$.

Also,

$N_{i,5}\,(ground, sex, 0, \boldsymbol{imm}) = 0$

(b) Some adults move from the home ground to the spawning ground.

$N_{i,5}\,(\boldsymbol{sp}, sex, age, \boldsymbol{mat}) = spawn\_p * N_{i,5}\,(\boldsymbol{hm}, sex, age, \boldsymbol{mat})$

$N_{i,5}\,(\boldsymbol{hm}, sex, age, \boldsymbol{mat})$ –= $spawn\_p * N_{i,5}\,(\boldsymbol{hm}, sex, age, \boldsymbol{mat})$

Stage 6: Spawning season: first half

$N_{i,6}\,(ground, sex, age, maturity) = N_{i,5}\,(ground, sex, age, maturity)$

(a) Natural and fishing mortality are applied to fish in the spawning ground.

For $age \geq 1$

$N_{i,6}\,(\boldsymbol{sp}, sex, age, maturity)$ *= $\exp[\,-t * (\,F(sex, age) + M(sex)\,)\,]$

where the fishing mortalities $F(sex, age)$ are calculated from the Baranov catch equation using the selectivities $sel\_sp(sex, age)$ and the spawning season catch. The time period t in this case is $0.5 * sp\_length$.

(b) Natural mortality is applied to fish in the corridor and the home ground.

For $age \geq 1$ and $ground\ \varepsilon\ \{\,\boldsymbol{cor}, \boldsymbol{hm}\,\}$

$N_{i,6}$ (*ground, sex, age, maturity*) *= exp[$-t$ * M(*sex*)]

(c) Larvae are created in the spawning grounds.

$larvae_i = R_i * virginR * fbio / [\ alpha + beta * fbio\ ]$

where *fbio* is the biomass of the females present in the spawning ground, *virginR* is the number of larvae needed to maintain deterministic equilibrium prior to fishing, and *alpha*, *beta* are the parameters of the Beverton-Holt stock-recruit relationship given by *steep*.

(d) Larvae are split by sex.

$larvae_i$(**male**) = *p_male* * $larvae_i$

$larvae_i$(***fem***) = (1 − *p_male*) * $larvae_i$

$N_{i,6}$ (***sp***, *sex, 0,* ***imm***) = $larvae_i$(*sex*)

Stage 7: Spawning season: second half

$N_{i,7}$ (*ground, sex, age, maturity*) = $N_{i,6}$ (*ground, sex, age, maturity*)

(a) Natural and fishing mortality are applied to fish in the spawning ground.

For $age \geq 1$

$N_{i,7}$ (***sp***, *sex, age, maturity*) *= exp[$-t$ * (F(*sex, age*) + M(*sex*))]

where the F(*sex, age*) and t are as in Stage 6.

(b) Natural mortality is applied to fish in the corridor and the home ground.

For $age \geq 1$ and *ground* ε { ***cor***, ***hm*** }

$N_{i,7}$ (*ground, sex, age, maturity*) *= exp[$-t$ * M(*sex*)]

Stage 8: End of fishing year

$N_{i,8}$ (*ground, sex, age, maturity*) = $N_{i,7}$ (*ground, sex, age, maturity*)

(a) Adults return from the spawning ground to their home ground.

$N_{i,8}$ (***hm***, *sex, age,* ***mat***) += $N_{i,8}$ (***sp***, *sex, age,* ***mat***)

$N_{i,8}$ (***sp***, *sex, age,* ***mat***) = 0

Appendix 2: The least squares and maximum likelihood estimators

Least squares

In general, a least squares estimate is a vector of parameter values which minimises a weighted sum of squared differences between the observations and the predicted values (as given by the vector of parameters when input into the model). The form of the sum of squares used for the least squares estimator in this paper is:

$$\sum_{k \varepsilon K} w_k \left[\ln(X_k) - \ln(P_k)\right]^2$$

where K indexes all observed values (biomass indices) and for $k \varepsilon K$, X_k is the kth observation, P_k is the kth predicted value, and w_k is the kth weight. The weights for each observation were calculated using the method described below.

Each observation has a "source code": observations with the same source code are theoretically derived from the same "source" (e.g., a series of trawl surveys—the source—giving as observations a time series of biomass indices). Let S be a subset of K which indexes observations with a particular source code, then for $s \varepsilon S$,

$$w_s = \frac{uy}{Wc_s^2}$$

where u is a specified source weight, y is the number of years for which there are observations from the source, c_s is a specified *c.v.*, and

$$W = \sum_{s \varepsilon S} \frac{1}{c_s^2}$$

Each observation also has a "q code": observations with the same q code are assumed to belong to a relative time series. Let Q be a subset of K which indexes observations with a particular q code, then for each $j \varepsilon Q$,

$$P_j = qT_j$$

where q is a proportionality constant and T_j is the predicted value before scaling. The value of q which minimises the sum of squares can be found analytically and is equal to:

$$\exp\left[\left(\frac{1}{\sum_{j \varepsilon Q} w_j}\right) \sum_{j \varepsilon Q} w_j \log\left(\frac{X_j}{T_j}\right)\right]$$

Maximum likelihood

In general, a maximum likelihood estimate is a vector of parameter values which gives the highest "probability" (or likelihood) of observing the actual observations.

Maximum likelihood estimators are usually obtained by minimising the negative log likelihood (which is equivalent to maximising the likelihood). To derive the log likelihood of the observations it is necessary to specify the statistical distribution of the observations.

For a time series of relative biomass indices, let K index the observations in the time series, then for $k \varepsilon K$ it is assumed for observation X_k that $X_k = qT_k\varepsilon_k$ where $\varepsilon_k \sim \mathrm{N}(1, c_k^2)$, q is a proportionality constant, T_k is the true value, and c_k is a given *c.v.*

All time series are considered to be mutually independent, so that the combined log likelihood is the sum of the individual log likelihoods. For a time series of biomass indices (indexed by K as above), the non-constant portion of the negative log likelihood is:

$$\sum_{k \varepsilon K}\left[\log(qT_k c_k) + \frac{1}{2c_k^2}\left(\frac{X_k}{qT_k} - 1\right)^2\right]$$

If the time series is relative (and hence q is unknown), the value of q which minimises the above equation can be found analytically and it is equal to:

$$\frac{\sqrt{m_1^2 + 4m_2} - m_1}{2}$$

where

$$m_1 = \frac{1}{n}\sum_{k \varepsilon K}\frac{X_k}{T_k c_k^2} \quad \text{and} \quad m_2 = \frac{1}{n}\sum_{k \varepsilon K}\frac{X_k^2}{T_k^2 c_k^2}$$

and n is the cardinality of K.

Appendix 3: MIAEL estimation of qB

Suppose that X is a relative biomass index obtained from a survey of a population with true biomass B. Let the proportionality constant be q and assume a multiplicative error structure:

$$X = q\,B\,e$$

where $\mathrm{E}(e) = 1$, $\mathrm{Var}(e) = \sigma^2$, and σ^2 is assumed known.

The index X can be considered as an estimator of $q\,B$. A *best p* MIAEL estimator (*see* Appendix 4) of $q\,B$ based on X can be derived as follows.

Let,

$$Y = p\,X + (1 - p)\,k$$

where k is the *best k* (or best constant) estimator of $q\ B\ \varepsilon\ [\ a, b\ ]$.

The *best p* estimator is found by deriving the p which minimises the Integrated Average Expected Loss of Y. Using a proportional squared error loss function gives:

$$k = a\,b\ln(\,b\,/\,a\,)\,/\,(\,b\ -a\,)$$

and, for $z = q\ B\ \varepsilon\ [\ a, b\ ]$ the proportional mean squared error of Y is:

$$\mathrm{PMSE}(Y, z) = [\,p^2\,\mathrm{Var}(X) + (\,p\,\mathrm{E}(X) + (1-p)\,k - z\,)^2\,]\,/\,z^2$$

Now, $\mathrm{E}(X) = q\ B$ and $\mathrm{Var}(X) = (\ q\ B\ \sigma\ )^2$ which depend only on $z = q\ B$. Hence, PMSE is equivalent to average PMSE.

On expansion and simplification,

$$\mathrm{PMSE}\,(Y, z) = [\,p^2\,(\sigma^2+1) - 2\,p\ + 1\,] - [2\,(1-p)^2\,k\,/\,z\,] + [(1-p)\,k\,/\,z]^2$$

Integrating over the interval $[a, b]$ with respect to z gives

$$\mathrm{IPMSE}\,(Y) = f(p, b) - f(p, a)$$

where

$$f(p, z) = [\,p^2\,(\sigma^2+1) - 2\,p\ + 1\,]\,z - [2\,(1-p)^2\,k\,]\ln(z) - (1-p)^2\,k^2\,/\,z$$

Finally, differentiating with respect to p, setting the result equal to zero, solving for p and substituting k gives:

$$p = h(a, b)\,/\,[\,(b-a)^2\,\sigma^2\ + h(a, b)\,]$$

where

$$h(a, b) = (b-a)^2\ - a\,b\ln(\,b\,/\,a\,)^2$$

Note, the reassuring properties of the expression for p: $\sigma^2 = 0$ gives $p = 1$ (and hence $Y = X$), and $\sigma^2 = \infty$ gives $p = 0$ (and hence $Y = k$).

Appendix 4: MIAEL estimation, the information index, and *best p* estimators

This appendix gives the reader who is unfamiliar with MIAEL estimation a detailed introduction to the motivation and definitions of the method. For further details on MIAEL estimation see Cordue (1998) and for *best p* estimators see Cordue (1995).

Decision theory and point estimation

Point estimation can be considered as a special case of decision theory (Wald 1950, Fergusson 1967, Berger 1985). In the general decision problem there is an unknown "state of nature" and a decision maker. The decision maker has to choose between a number of alternative actions, each of which will result in a "loss" depending on the true state of nature. The decision maker may conduct an experiment (i.e., observe some random variable whose distribution (hopefully) depends on the state of nature) in order to help them decide on the "best" action.

For example, a classic statistical problem is estimating the probability of getting "heads" from the single toss of a given coin (a special case of a Bernoulli experiment). The unknown "state of nature" is the probability of getting "heads". The "actions" available to the decision maker are their possible choices for the estimate: any real number from 0 to 1 inclusive. The "loss" in this case is estimation loss; presumably the further away that the estimate is from the true value, the greater the loss. The usual experiment conducted by the decision maker is to toss the coin n times, and record the total number of times that "heads" occurs. On the basis of this observation, they choose their estimate (action).

In more precise terms, for the general decision problem, there is an unknown state of nature θ contained in a parameter space Θ. The decision maker can observe a random variable X which has observable values in Obs(X) (with a generic observation denoted by x), and probability density function $p(x \mid \theta)$. An action $a \in A$ must be chosen, and this will result in a non-negative loss given by the function $L : \Theta \times A \rightarrow \mathrm{R}$. The solution to the decision problem is to find a decision rule $d : \mathrm{Obs}(X) \rightarrow A$, which minimises (in some sense) the expected loss $\mathrm{E}_{X \mid \Theta}[\ L(\theta,d(X))\ ]$. ($\mathrm{E}_{X \mid \Theta}$ denotes the expectation with respect to X assuming that θ is the true state of nature.) The expected loss $\mathrm{E}_{X \mid \Theta}[\ L(\theta,d(X))\ ]$ is called the risk function of d, and will be denoted here by $R(\theta,d)$.

In the general point estimation problem, estimating $g(\theta)$ for some given function g, the actions consist of the possible choices for the estimate, so that $d(X)$ is simply an estimator (and for $x \in \mathrm{Obs}(X)$, $d(x)$ is an estimate). The loss function will then be a function of $g(\theta)$ and $d(X)$ and should in some sense measure the "distance" between them, with increasing loss as the "distance" increases. An estimator $d(X)$ which in some sense minimises $R(\theta,d)$, is then minimising the expected "distance" between $g(\theta)$ and $d(X)$. For example, a commonly used loss function is squared error $[\ g(\theta) - d(X)]^2$, which results in mean squared error as a risk function. An optimal estimator in this case, then, minimises (in some sense) mean squared error.

Continuing with the coin tossing example, rather than estimating θ, the probability of getting "heads", it may be desired to estimate a function of θ, say θ^2. Also, the decision rule d, might be "divide the total number of heads observed by the number of trials, and square the result". That is, $d(X) = (X/n)^2$ where X is the total number of heads observed in

n trials. If the loss function L is squared error, then $L(\theta,d(X)) = [\ \theta^2 - X^2/n^2\ ]^2$. The risk function of the estimator $(X/n)^2$ is $E_{X|\theta}[\ \theta^2 - X^2/n^2\ ]^2$.

The general formulation is intuitively appealing, but there is the difficulty of deciding in what sense the risk function is to be minimised. In general there will not be an estimator with minimum risk for all values of θ. (Consider for example, estimating $\theta \in [0,1]$ with a squared error loss function. For any constant $k \in [0,1]$, $d(X) = k$ has zero risk when $\theta = k$, hence there cannot be an estimator of θ with uniformly minimum mean squared error.) Three main approaches have been used: imposition of a special property to form a "class" of estimators within which uniformly minimum risk is sought (e.g., considering only unbiased estimators); minimising the maximum risk (minimax estimation); and minimising a weighted average risk (e.g., Bayes estimation, where the weighting is given by the prior distribution of Θ — although, of course, Bayes estimation can be developed more simply and independently of the approach described here). MIAEL estimation is related to Bayes estimation, but its formulation differs because the averaging is done over $g(\Theta)$ rather than Θ (note, $g(\Theta) = \{\ g(\theta)\ |\ \theta \in \Theta\}$).

MIAEL estimation

The main idea behind MIAEL estimation is that since $g(\theta)$ is the object of interest, the minimisation of estimation risk should be done in the $g(\theta)$ domain (i.e., within $g(\Theta)$ rather than Θ). Also, a uniform weighting is used in the integration of risk (across $g(\Theta)$) because, inasmuch as $g(\theta)$ is unknown, there is little reason to require preferential estimator performance in any particular region of $g(\Theta)$. The aim is to minimise the "average" risk, given the estimation losses encapsulated in the specified loss function.

The integrated average expected loss of $d(X)$ when estimating $g(\theta)$ with loss function L is

$$I[\ d(X)\ ] = \int_{z \in g(\Theta)} \left[\frac{\displaystyle\int_{\phi \in g^{-1}(z)} R(\phi, d)\, d\phi}{\displaystyle\int_{\phi \in g^{-1}(z)} d\phi} \right] dz$$

where

$$\forall\ \theta \in \Theta \quad g^{-1}\ (g(\theta)) = \{\ \phi \mid \phi \in \Theta \text{ and } g(\phi) = g(\theta)\}$$

and if $g^{-1}(z)$ is finite, then integration over $g^{-1}(z)$ is interpreted as simple summation. If $d \in D$ is such that for every $d' \in D$, $I\,[d'(X)] \geq I\,[d(X)]$ then $d(X)$ is a MIAEL estimator within the class D.

This definition requires some clarification. In the MIAEL acronym, "EL" denotes Expected Loss (expectation over X of the loss function). The "A" is Averaging of Expected Loss for each point in $g(\Theta)$. For each $z \in g(\Theta)$, the Average Expected Loss is given by the ratio of the integrals in the definition of $I[d(X)]$. Since $z \in g(\Theta)$ and

$\phi \in g^{-1}(z)$, there exists $\theta \in \Theta : g(\phi) = g(\theta)$. The denominator in the ratio is a "count" of the number of points in $g^{-1}(z)$ (explicitly if $g^{-1}(z)$ is finite) and the numerator is the "sum" of the expected losses. Note, that if g is 1 to 1 then $g^{-1}(z)$ contains only a single point, and the "A" is redundant.

In the coin tossing example, if θ or θ^2 were being estimated, then g is 1 to 1 (as $\theta \in [0,1]$) and no averaging of expected losses occurs. However, if $g(\theta) = \theta(1 - \theta)$ was being estimated, then $g(\Theta) = [0,1/4]$ and for any $z \in [0,1/4]$, $g^{-1}(z) = \{ t_0, 1 - t_0 \}$ where t_0 is a solution to $z = \theta(1 - \theta)$. Hence, for every point in $g(\Theta)$, the ratio of the integrals is an average of exactly two expected losses. Note, that there is no guarantee that $R(t_0,d) = R(1 - t_0, d)$. In general, the risk of a decision rule is a function of θ, not $g(\theta)$.

Returning to the general case, note that $I[d(X)]$ does not necessarily exist (it may be infinite) and hence for some classes a MIAEL estimator may not exist. If a MIAEL estimator does exist it may not be unique. However, in almost every practical fisheries application there will be sufficient ancillary information available to allow θ and $g(\theta)$ to be bounded. In that case, MIAEL estimators within many general classes will exist and be unique within the class. Also, in some circumstances, a global MIAEL estimator will exist (*see* theorems 1–3 in Cordue 1998).

An information index

Point estimates by themselves are sometimes not particularly useful to fishery managers. It is generally desirable to include some measure of the uncertainty of an estimate. Usually, this is done by providing a confidence interval at some high level of confidence (traditionally 95%, more recently 90%). The confidence interval approach is of limited value in some fisheries applications, particularly in "risk" analysis, where confidence intervals on "risk" (if they were ever calculated) would often include the interval [0,1]. If MIAEL estimation is used then a natural measure of estimator uncertainty can be provided by comparing the relative performance of the MIAEL estimator which uses the observations and the MIAEL estimator which does not.

Let D be a class of estimators (based on X, estimating $g(\theta)$, with loss function L), and let the information index of $d \in D$ be defined as

$$Info\,[\,d\,] = 1 - \frac{\mathrm{I}\,[\,\mathrm{d(X)}\,]}{\mathrm{I}\,[\,\mathrm{K}\,]}$$

where K (called the *best k* estimator) is the MIAEL estimator of $g(\theta)$ (under loss function L) before the experiment is observed (i.e., when no observations are available). If D contains a MIAEL estimator $M(X)$, then for every $d \in D$, $Info(M) \geq Info(d)$.

If X has a distribution which does not depend on θ then $Info[d] \leq 0$ (since $I[d(X)] \geq I[K]$). Also, as estimation losses cannot be negative, for every $d \in D$ $I[d(X)] \geq 0$. Hence, an information index (as defined) is always less than 1, and equals 1 if and only if $I[d(X)] = 0$. Note that provided $K \in D$, $Info[M]$ is always in the interval [0,1]. The MIAEL estimator K can easily be found for a squared error loss function (and other simple loss functions). In the case when $g(\Theta) = [a,b]$ with a squared error loss function,

$K = (a + b)/2$. (For K under proportional squared error, *see* Appendix 3.) Under fairly general conditions, it is always the case that $K = k$ for some $k \in g(\Theta)$ (*see* theorem 1 in Cordue 1998).

The *best p* estimator

Finding a MIAEL estimator from the class of all estimators is often difficult or impossible. To find a MIAEL estimator for a particular problem it is often necessary to construct a restricted class of estimators and determine the MIAEL estimator within the class. One way to construct a class of estimators is to build it around a standard estimator, derived from a method such as least squares or maximum likelihood. This is how "*best p*" estimators are constructed; they are MIAEL estimators within particular classes of estimators built from a "base" estimator.

Continuing with the notation above, let

$$P = \{\, p\, d(X) + (1 - p)\, K \mid p \in \mathrm{R} \,\}$$

for some estimator $d(X)$ where K is the *best k* estimator. The MIAEL estimator in the class P is called a *best p* estimator; it is derived from the base estimator $d(X)$. Note, both $d(X)$ and K are in the class P, and that because $K \in P$ it follows that the information index of the MIAEL estimator is between 0 and 1.