NEW ZEALAND METEOROLOGICAL SERVICE

TECHNICAL NOTE NO. 180

FREQUENCIES OF DAILY RAINFALL OF SPECIFIED AMOUNTS AT NANDI AND LAUTHALA BAY, FIJI

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Issued by:

The Director,
New Zealand Meteorological Service,
P.O. Box 722,
WELLINGTON.

14 March 1969

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Abstract

Empirical equations are derived relating daily rainfall amount and frequency. Nomograms based on the equations are given, from which the "design rainfall" can be estimated at specified probability levels and for specified return periods.

The "probable maximum" rainfall is discussed, and estimates are made. Improvements to the existing instrumentation are suggested in order to ensure accurate recording of exceptional rainfalls.

Introduction

Estimates of the probability of specified amounts of rain in a day in a specified period of years are of obvious importance to engineers, agriculturists, insurance actuaries and others. The present paper presents a simple method of obtaining such estimates for Nandi Airport (17°45'S 177°27'E) and Lauthala Bay (18°09'S 178°27'E), two representative stations in Fiji, for which some 25 years of reliable daily observations became available at the end of 1967.

The estimates are based on empirical equations fitted to the whole of the data and checked for significance by means of the χ^2 test. This method is considered preferable to other methods utilising only the annual extremes, and assuming a type of distribution to which the data do not necessarily conform.

Annual extremes of rainfall are not very reliable where there is a possibility of the gauge overflowing, as is the case in Fiji.

The problem of specifying "probable maximum" rainfall is discussed in the light of observations of extreme rainfalls in Fiji, and empirical methods developed by other workers are used to compute reasonable values of these maxima.

As a corollary to this discussion, shortcomings in the existing instrumentation are revealed. It is shown that existing raingauges in Fiji are inadequate to record the heaviest possible rainfalls, and suggestions are made for their improvement.

Data

9150 daily rainfall measurements were available for Nandi Airport, covering an aggregate of about 25 years between January 1942 and December 1967. Breaks in the record amounted to just over 12 months.

9100 similar measurements were available for Lauthala Bay, covering the 25 years January 1943 to December 1967, apart from one break of a month.

Details of the gauges and the observation sites are given in Appendix I.

Analysis

Class frequencies of rainfall in specified ranges were first tabulated. Cumulative frequencies (i.e. the frequencies with which specified amounts were equalled or exceeded) were derived by successive subtraction of the class frequencies from the total number of observations.

Following the results given for Kilmarnock by Brooks and Carruthers (1953, p.120), it was expected that an equation of the form

$$\log F_R = a + bR$$

might fit the data, where a, b are constants, F_R is the cumulative frequency of daily rainfalls $\geq R$.

Accordingly, the values of F_R were plotted on semi-logarithmic paper against corresponding values of R (Fig. 1). Contrary to expectation, the points do not lie near a straight line, but appeared to lie near a smooth curve. A fit to an equation of the general form

$$\log F_R = a + bR^c$$

where c is a constant, was tried.

In order to simplify the arithmetic, the cumulative frequencies F_R were divided by the total number of observations, yielding the relative cumulative frequencies, denoted by ϕ_R.

The following equations were obtained using methods detailed in Brooks and Carruthers (1953):(Unless otherwise indicated, logarithms are to base 10).

Nandi Airport

$$\log \phi_R = - 0.462 - 0.745R^{0.678} \quad \ldots\ldots \quad (1)$$

Lauthala Bay

$$\log \phi_R = - 0.211 - 0.821R^{0.581} \quad \ldots\ldots \quad (2)$$

Computed frequencies for the period of observation are given by the following equations:

Nandi Airport

$$\log (9150\, \phi_R) = \log 9150 - 0.462 - 0.745R^{0.678} \ldots\ldots (3)$$

Lauthala Bay

$$\log (9100\, \phi_R) = \log 9100 - 0.211 - 0.821R^{0.581} \ldots\ldots (4)$$

Annual average frequencies are given by the following equations:

Nandi Airport

$$\log (365\, \phi_R) = \log 365 - 0.462 - 0.745R^{0.678} \quad \ldots\ldots \quad (5)$$

Lauthala Bay

$$\log (365\, \phi_R) = \log 365 - 0.211 - 0.821R^{0.581} \quad \ldots\ldots \quad (6)$$

Computed cumulative frequencies for the period of observations derived from Equation (3) are shown in Appendix II (Table 1A), together with class frequencies obtained by successive subtraction of the computed cumulative frequencies, together with the corresponding observed values for Nandi Airport.

Similar information for Lauthala Bay is shown in Appendix III (Table 1B).

In each case the computed and observed frequencies are in good agreement, and the χ^2 test confirms that the data are satisfactorily represented by the equations.

The observed annual average cumulative frequencies are shown as points in Figs 2 and 3. The frequency scale is logarithmic, and the scale of R is proportional to $R^{0.678}$ in Fig. 2 (Nandi Airport) and $R^{0.581}$ in Fig. 3 (Lauthala Bay). Equations (5) and (6) are therefore represented by straight lines.

The points representing the frequencies of 0.01 in. and 0.04 in. are shown, although these values were not used in the computations. Their deviation above the computed curve is regarded as due to asymmetry of the distribution of rainfall in the range 0.00-0.19 in.

The points representing the frequencies of some of the heaviest rainfalls also deviate from the computed curves. These deviations are ascribed to the fact that the points represent only a few observations, and are not considered to be significant.

Application

The graphs, Figs 2 and 3, may be used to determine the average number of occasions per year of daily rainfalls equal to or exceeding specified amounts.

This information is expressed in a more useful form by taking the reciprocal of the number of occasions per year; this gives the average period in years elapsing between successive occurrences of the specified rainfall, i.e. the return period.

The information given by Figs 2 and 3 is of limited usefulness to the engineer or to the insurance actuary, who each require to know what rainfall they must allow for in a given period in order to reduce their risk to a specified level.

It is evident that in some return periods the "expected" rainfall given by Figs 2 and 3 will not occur, while in others it may occur more than once.

We can assist the engineer and insurance actuary by indicating what "design" rainfall to allow for at specified probability levels and for specified return periods.

The highest daily rainfall of even one year may be regarded as a "rare event" - one out of 365. If the expected frequency of such an event is x, then the probability of its non-occurrence is e^{-x} (the first term of Poisson's series). The probability of at least one such occurrence is therefore $1 - e^{-x}$.

The "design" rainfall R_d may be computed as follows:

(1) Equate the expression $1-e^{-x}$ to the required probability and evaluate x.

(2) Divide this value of x by P, the required return period.

(3) Substitute this value of x/P for 365 ϕ_R in Equation (5) or (6) as appropriate, and evaluate R. This is the required value of R_d.

In order to avoid unnecessary repetition of the computations, the nomograms Figs 4 and 5 have been constructed. They are based on Figs 2 and 3 respectively by the substitution of a scale of return period for the scale of 365 ϕ_R, and the superimpostion of a probability grid. The "expected" curve is that derived from Equation (5) or (6).

The probability grid is constructed as follows:

> For a selected return period, compute as shown above the values of R_d for the required series of values of the probability. Mark off on the isopleth representing the return period the intersections of the ordinates through the computed values of R_d. Draw through these points straight lines parallel to the "expected" curve.

Curves for various probability levels between 2% and 98% are shown in the diagrams. The "expected" curve corresponds to a probability level of 63.2%. This is the probability of occurrence of a rainfall whose expected or average frequency in a given return period is 1.

Selected values of R_d estimated from Figs 4 and 5 are given in Appendix IV, Table 2.

These values may be compared with 24-hour maxima computed from Chow's Equation, and appended to Table 2:

$$X_T = \bar{X} + S_X K$$

(See Lockwood (1967) quoting Chow (1951) and Weiss (1955))

where X_T = magnitude of item with return period T

$\bar{X}$ = mean of the extreme values

S_X = standard deviation of the series of extreme values

K = frequency factor depending on return period and length of record

The following interpolated values of K were used, for length of record 25 years:

Return Period yrs	K
10	1.5829
100	3.7445

According to Lockwood (1967) Chow's Equation gives results very similar to those obtained by Gumbel's method (Gumbel 1958).

It will be seen that these values lie near our "expected" or 50% values.

Probable Maximum Daily Rainfall

It is emphasised that the "design" rainfall estimated by the methods described above is the rainfall which will be equalled or exceeded at least once in the specified return period, and at the specified probability level.

The question arises, and is often asked, "What is the probable maximum rainfall?". Such a question involves a contradiction of terms. The maximum rainfall is a unique event, to which we can hardly assign a definite probability.

We can however estimate its magnitude, using non-statistical arguments.

Court and Salmela (1963) suggest that the "improbable extreme" rainfall for the world, of duration t minutes, is given by the formula

$$R_E = 2t^{\frac{1}{2}}$$

Putting t = 1440 (the number of minutes in a day) we obtain an estimate of nearly 76 in. for the "improbable extreme" daily rainfall for the world.

This figure seems impossibly high, yet it was nearly equalled on the 15/16th March 1952, when a 24-hour fall of 73.62 in. was recorded at the village of Cilaos at an elevation of 3937 feet on the island of La Réunion in the Indian Ocean, during a tropical cyclone. In a discussion of this phenomenal rainfall, Paulhus (1965) points out that La Réunion is extremely mountainous, with elevations over 10,000 feet, and that the heavy rainfalls associated with tropical cyclones are greatly intensified by orographic influences.

Fiji lies in almost the same latitude as La Réunion (about 18°S compared with 21°S) and in a similar maritime location. Although the mountains are much lower (highest peak 4341 feet) the main island of Viti Levu has similar steep slopes and narrowing valleys. In view of the fact that daily rainfalls of around 40 in. have occurred in Fiji near sea level (see below) there seems little doubt that amounts approaching the Cilaos figure could occur in the high country during a hurricane.

Lockwood (1967) quotes Hershfield (1965) and other U.S. Weather Bureau workers (1961) as suggesting that PMP (Probable Maximum Precipitation) values may be estimated by setting the frequency factor, K, in Chow's Equation (see above) to 15, which is a maximum based on an empirical analysis of data from widely scattered countries. He does not define in this paper what is meant by "probable", but in 1968 he states that many PMP estimates seem to have return periods varying between 10^4 and 10^8 years.

Leaving aside for the moment the definition of "probable" in this context, we find that PMP values calculated in this way for Nandi Airport and Lauthala Bay are 37.44 in. and 44.08 in. respectively.

These values may be compared with the highest daily rainfalls ever recorded in Fiji since 1886, when records began at Government House, Suva.

46 in.	8 August 1906. Reported by the warship Pegasus in Suva Harbour, but not officially accepted. Derrick (1957).
37 in.	8 August 1906. Reported by a "reliable private observer" in Suva. The gauge overflowed at 10 p.m. and 2 a.m. At 6 a.m. it was again full. Capacity 12½ in. Derrick (1957).
36.5 in.	21 December 1938. At Navai, near Mt. Victoria, N. Viti Levu. The gauge overflowed. Estimated fall over 40 in. Derrick (1957).
35.00 in.	21 February 1931. At Rarawai Mill.
27.56 in.	16 November 1963. At Salialevu.
27.55 in.	8 February 1965. At Nandarivatu.
26.60 in.	8 April 1940. At Rambe.
26.50 in.	8 August 1906. At Government House, Suva. (See first two entries). The gauge, of capacity 13 in., overflowed twice. The rain fell during an electrical storm lasting 13 hours.

In view of the foregoing, it appears that daily rainfalls of between 40 and 50 in. can occur somewhere in Fiji (though not necessarily at Nandi Airport or Lauthala Bay) with an expectancy of once or twice in a long lifetime. Falls approaching 70 or 75 in. might occur; though such an event would be very surprising, we could not say that it is impossible.

Seasonal Incidence of Heavy Rainfalls

It has been suggested by Lockwood (1967 and 1968) that extreme daily rainfalls are likely to be associated with highly organised storms such as tropical storms. Since Fiji is liable to hurricanes during the six months 1 November to 30 April, we would expect the annual rainfall extremes to occur in that season if Lockwood's suggestion is correct.

Tabulated below are the numbers of occasions that the annual extreme daily rainfall occurred in each month of the year at Nandi Airport and Lauthala Bay.

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Nandi Airport	3	4	5	2	2	1	2	1	1	1	-	3
Lauthala Bay	3	8	-	4	-	3	1	-	-	1	3	2
Totals	6	12	5	6	2	4	3	1	1	2	3	5

Totals

Hurricane season November-April	37
Dry season May-October	13

These figures lend some support to our expectation, but it is to be noted that the annual extreme can occur at both stations in any season, and indeed in almost any month of the year (no doubt the gaps would be filled in a longer period of observation).

It is pertinent also to note that the highest daily rainfall in Suva, quoted above, or perhaps in all Fiji if we accept the warship's report of 46 in. occurred in August, associated not with a tropical storm, but with an electrical storm.

It seems reasonable therefore to conclude that while extreme daily rainfalls are more likely to occur during the hurricane season, they could occur in almost any month, and there is no essential association with tropical storms.

These remarks do not apply to rainfalls causing the most serious flooding. These seem to be almost entirely associated with tropical storms, but are usually spread over a period of some days and over a large area. They are therefore outside the scope of the present discussion.

Conclusions

24-hour "design" rainfalls can be estimated from Figs 4 or 5 as appropriate, or by computation from the equations. The selected values given in Appendix IV, Table 2, may be useful. These results apply strictly to Nandi Airport or Lauthala Bay.

However, these stations may be regarded as representative of the "dry" and "wet" zones of Fiji respectively, as defined by Harris (1963), and the results may reasonably be applied to other places within these zones.

It seems desirable to extend the analysis to other rainfall stations in Fiji, some of which have fairly long records. It seems possible, if a sufficient number of such analyses are made, that some functional relationship may be found between the annual average rainfall and the constants of the equations, permitting development of a more generalised equation.

We have quoted a number of the highest daily rainfalls reported in Fiji. On a number of occasions it was noted that the gauge had overflowed, so that it was not possible to record the true amount. Such overflowing may well be a common occurrence.

The capacity of the inner can of the 5 in. gauge at Nandi Airport is only 7.6 in.,and that of the 8 in. gauge at Lauthala Bay is about $11\frac{1}{2}$ in. It is certain that a similar situation exists at the other Fiji stations.

Court and Salmela (1963) recommended that raingauges should be capable of measuring 2 in. in one minute and a 72 in. accumulation in 24 hours, if further world records are to be observed. It is considered that this recommendation deserves serious consideration.

The Dines Tilting Siphon recording raingauges could not in fact record the exceptional rainfalls under discussion. It was shown by Matthews (1938) that this instrument sustains losses at each siphoning cycle which are a quadratic function of the rate of rainfall, and that the instrument fails altogether when the rate of rainfall exceeds about 12 in. per hour; the siphon does not break, but continues to dribble the water out as fast as it enters the float chamber.

It is suggested that the manual raingauge could be supplemented by a second gauge, of similar pattern, but fitted with a funnel of surface area say 1/10 that of the normal gauge (diameter 1.581 in. for the 5 in. gauge, 2.530 in. for the 8 in. gauge). The normal measuring glass could be used, the readings to be multiplied by 10.

A similar provision could be made for the Tilting Siphon recorder, which would then record 50 mm of rain before siphoning.

This instrument could also be improved by a modification whereby the rain entering the funnel during the siphoning cycle is diverted to waste instead of entering the float chamber. The loss during the siphoning cycle would then be a linear function of the rate of rainfall, which can usually be estimated from the record. The siphon would also break when the float chamber is empty, enabling the record to continue.

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APPENDIX I

The Raingauges and their Exposure

Nandi Airport

Lat. 17°45'S	Long. 177°27'E
Height above MSL	49 ft
Gauge Diameter	5 in. (8 in. up to 7 February 1956)

Height of rim above ground 1 ft

The original recording site was about 200 yards south of the E-W runway. From January 1965 onwards, the records refer to a site near the Operations Building about 200 yds north of the E-W runway, and 500 yds NE of the original site, and at the same height above MSL. The records are regarded as homogeneous.

Lauthala Bay

Lat. 18°09'S	Long. 178°27'E
Height above MSL	19 ft
Gauge Diameter	8 in.

Height of rim above ground 1 ft

The gauge is situated in the Meteorological Enclosure near the former RNZAF base at Lauthala Bay, now part of the campus of the University of the South Pacific.

APPENDIX II

Table 1A. Nandi Airport. Observed and Computed Frequencies* of Rainfall compared by the χ^2 test

R in.	Cum. Freq. Comp'd	Cum. Freq. Obs'd	Range in.	Class Freq. Comp'd c	Class Freq. Obs'd o	Diff. o - c	$\frac{(o-c)^2}{c}$
.00	9150	9150	under 20	7376	7409	+33	.15
.20	1774	1741	.20/.39	519	523	+ 4	.03
.40	1255	1218	.40/.59	317	281	-36	4.09
.60	938	937	.60/.79	217	208	- 9	.37
.80	721	729	.80/.99	154	146	- 8	.42
1.00	567	583	1/1.99	365	378	+13	.46
2.00	202	205	2/2.99	117	120	+ 3	.08
3.00	85.0	85	3/3.99	46.0	44	- 2	.10
4.00	39.0	41	4/4.99	20.0	23	+ 3	.45
5.00	19.0	18	5/5.99	9.3	8	-1.3	.18
6.00	9.7	10	6 and over	9.7	10	+0.3	.01
				Sum (= χ^2)			6.34
7.00	5.1	5					
8.00	2.8	4					
9.00	1.6	3					
10.0	0.9	1					

Degrees of freedom = 7.

With 7 degrees of freedom, a χ^2 value of 6.34 is near the 50% level of significance. Equation (3) therefore represents the data satisfactorily.

* Computed from Equation (3)

APPENDIX III

Table 1B. Lauthala Bay. Observed and Computed Frequencies* of Rainfall compared by the χ^2 test

R in.	Cum. Freq. Comp'd	Cum. Freq. Obs'd	Range in.	Class Freq. Comp'd c	Class Freq. Obs'd o	Diff. o - c	$\frac{(o-c)^2}{c}$
.00	9100	9100	under 20	6438	6457	+19	.06
.20	2662	2643	.20/.39	818	796	-22	.59
.40	1844	1847	.40/.59	471	453	-18	.69
.60	1373	1394	.60/.79	310	320	+10	.32
.80	1063	1074	.80/.99	219	233	+14	.90
1.00	844	841	1/1.99	514	515	+ 1	.00
2.00	330	326	2/2.99	174	174	0	.00
3.00	156	152	3/3.99	74.8	72	-2.8	.10
4.00	81.2	80	4/4.99	36.0	36	0	.00
5.00	45.2	44	5/5.99	18.8	15	-3.8	.77
6.00	26.4	29	6/6.99	10.4	12	+1.6	.25
7.00	16.0	17	7/7.99	6.0	7	+1.0	.17
8.00	10.0	10	8 and above	10.0	10	0	.00
					Sum (= χ^2)		3.85
9.00	6.4	6					
10.00	4.1	4					
11.00	2.8	4					
12.00	1.9	2					
13.00	1.3	1					

Degrees of freedom = 9.

With 9 degrees of freedom, a χ^2 value of 3.85 approaches the 90% level of significance. Equation (4) therefore represents the data satisfactorily.

* Computed from Equation (4)

APPENDIX IV

Table 2. Daily Rainfalls with Specified Probability of Occurrence in Specified Return Periods Estimated from Figs 4 and 5

Prob. Level %	Return Period: Nandi Airport 1	10	100	Lauthala Bay 1	10	100
98	2.8	6.0	9.9	3.7	8.0	13.7
95	3.1	6.3	10.2	4.1	8.6	14.4
90	3.4	6.8	10.9	4.6	9.2	15.2
75	4.1	7.7	11.8	5.5	10.6	16.9
*63.2	4.6	8.2	12.5	6.0	11.2	17.7
50	5.1	8.9	13.1	6.9	12.2	18.9
25	6.3	10.2	14.8	8.8	14.7	21.7
10	8.1	12.3	17.0	11.1	17.6	25.2
5	9.3	13.7	18.6	13.0	19.9	27.8
2	11.1	15.6	20.8	15.6	22.9	31.2
Computed from Chow's Equation (see text)		8.8	13.4		11.3	16.6

* Expectation = 1.

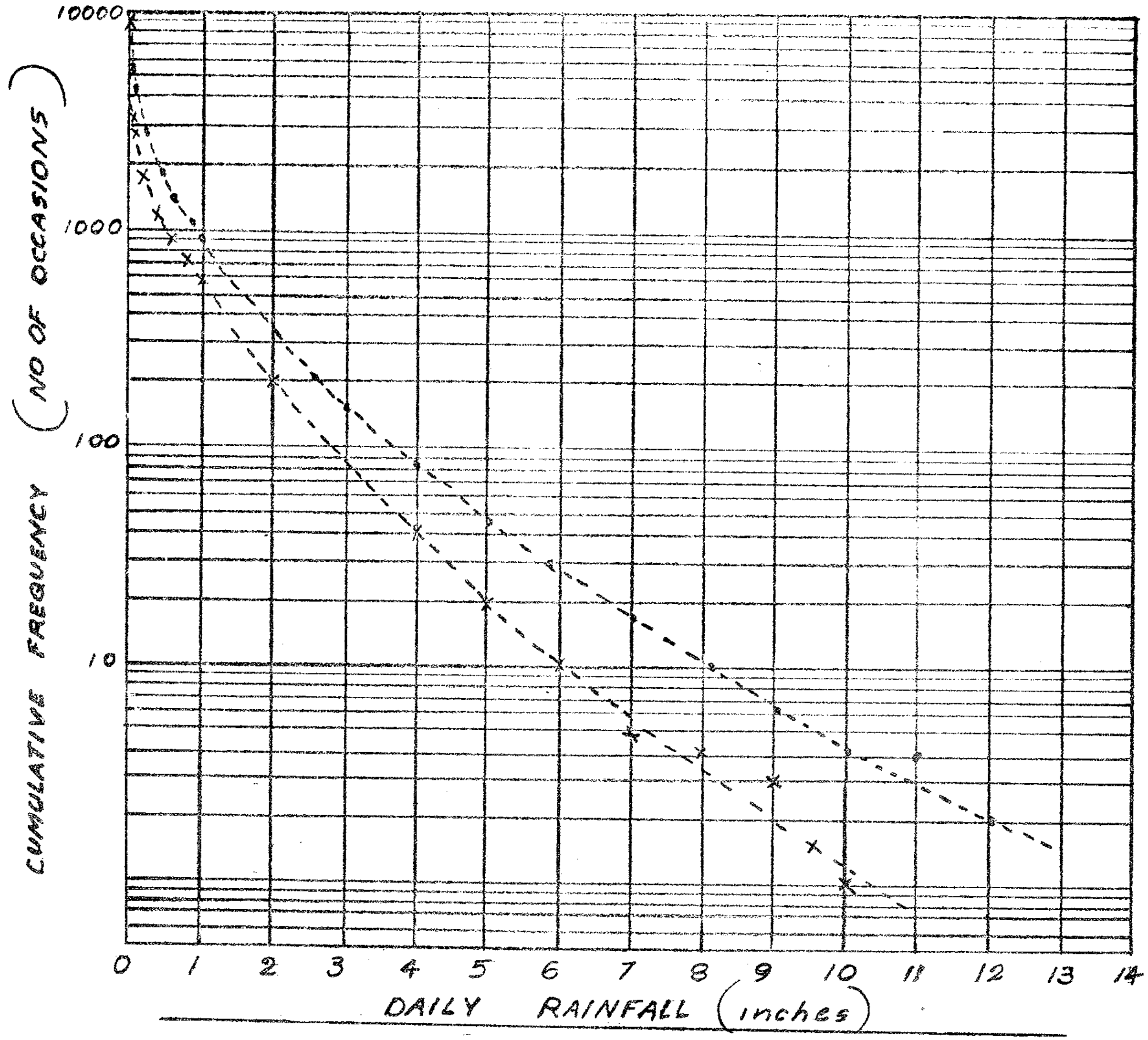

FIG I CUMULATIVE FREQUENCIES OF DAILY RAINFALL AT LAUTHALA BAY AND NANDI AIRPORT

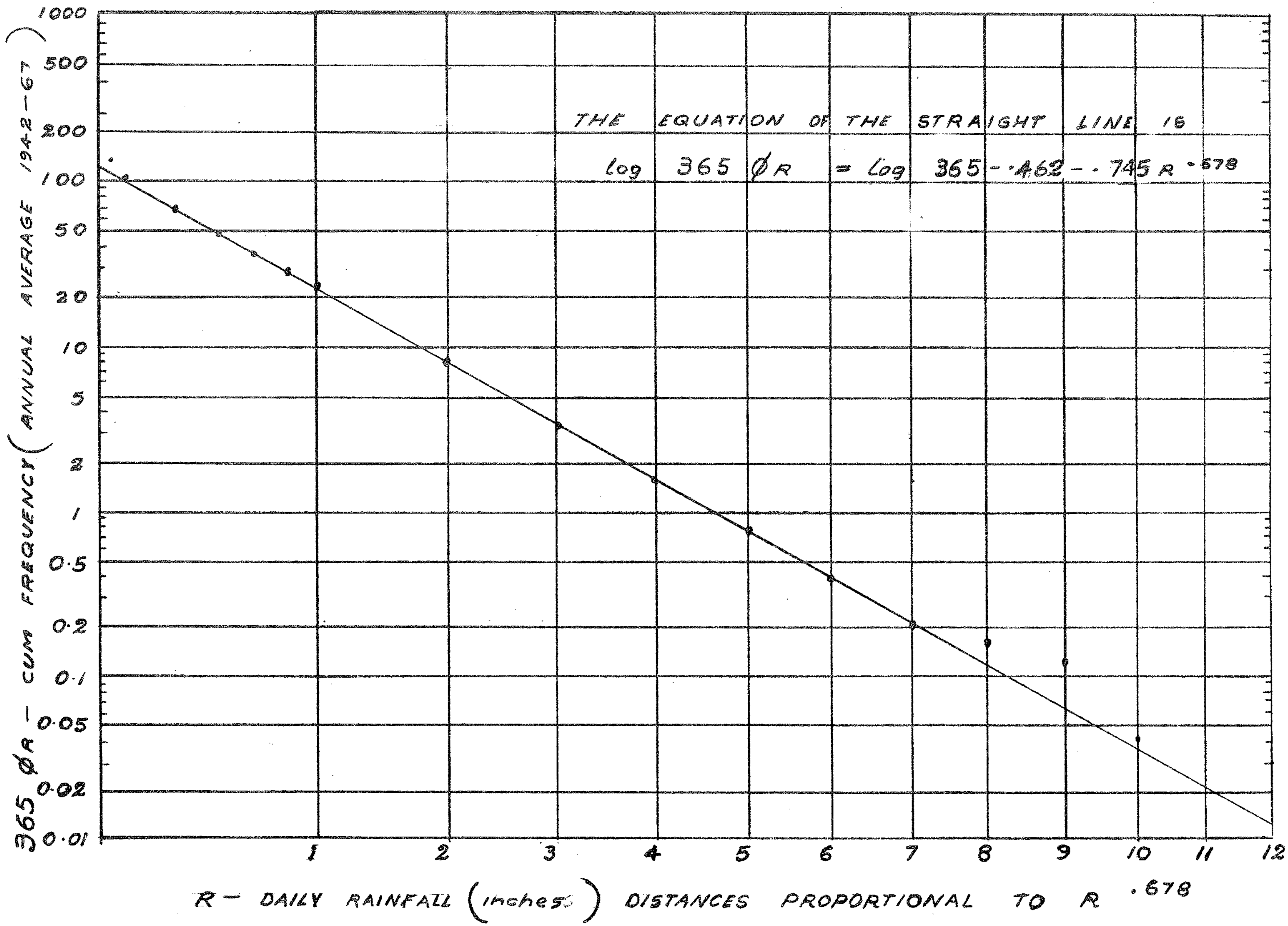

FIG 2 ANNUAL CUMULATIVE FREQUENCIES OF DAILY RAINFALL OF SPECIFIED AMOUNTS AT <u>NANDI AIRPORT</u>

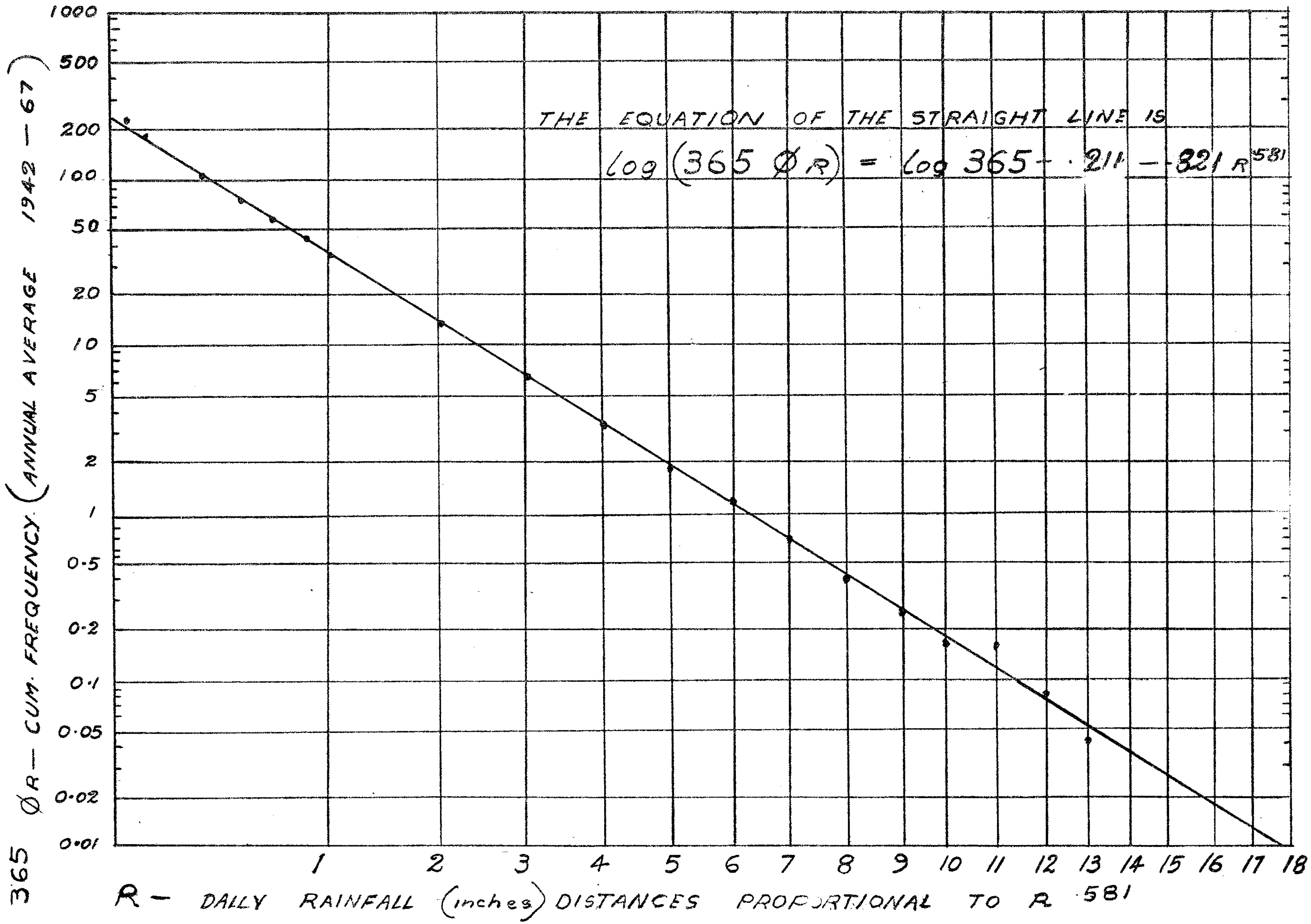

FIG 3 ANNUAL CUMULATIVE FREQUENCIES OF DAILY RAINFALL OF SPECIFIED AMOUNTS AT LAUTHALA BAY

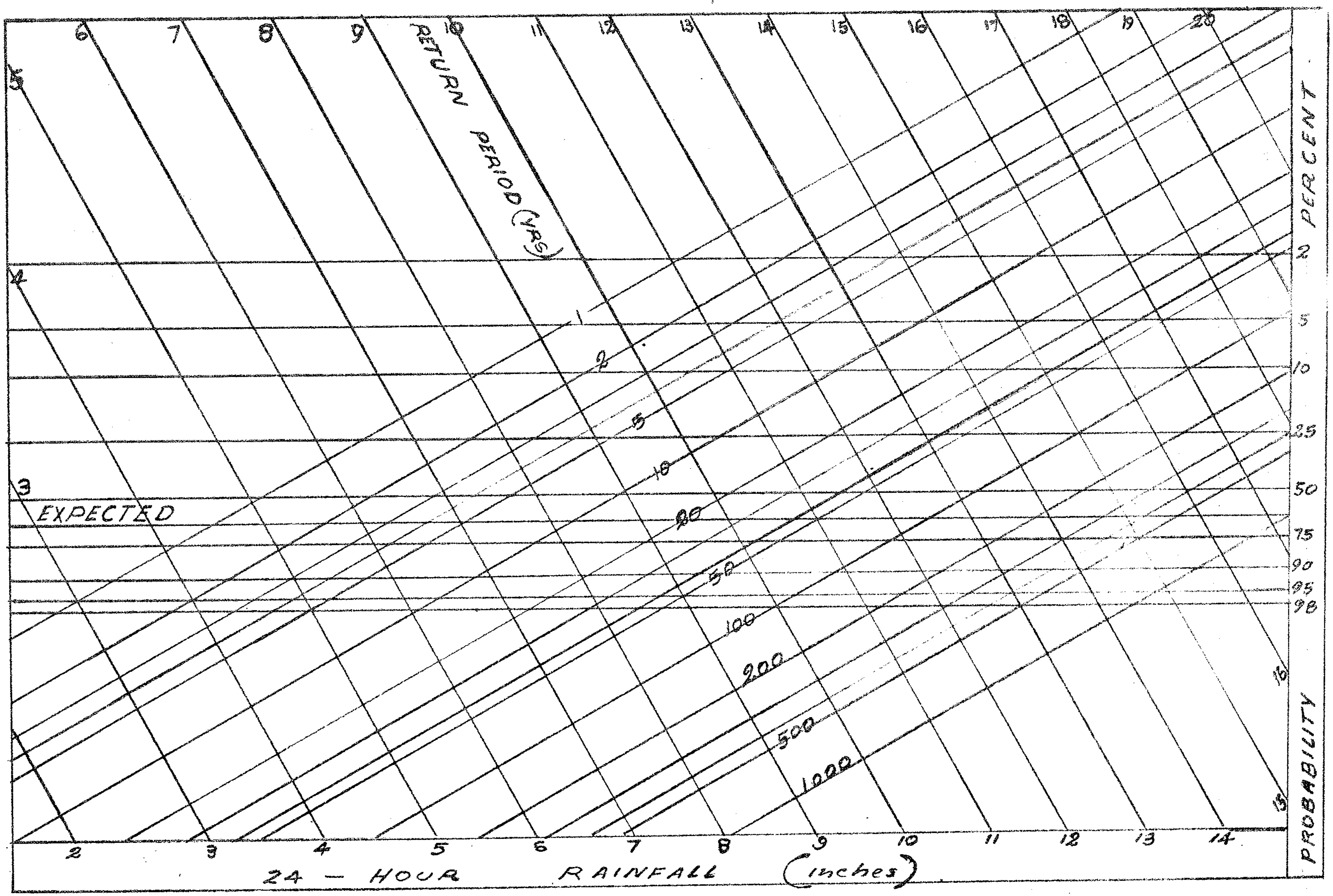

FIG 4 NANDI AIRPORT – NOMOGRAM FOR OBTAINING PROBABILITY OF SPECIFIED AMOUNTS OF DAILY RAINFALL IN SPECIFIED RETURN PERIODS.

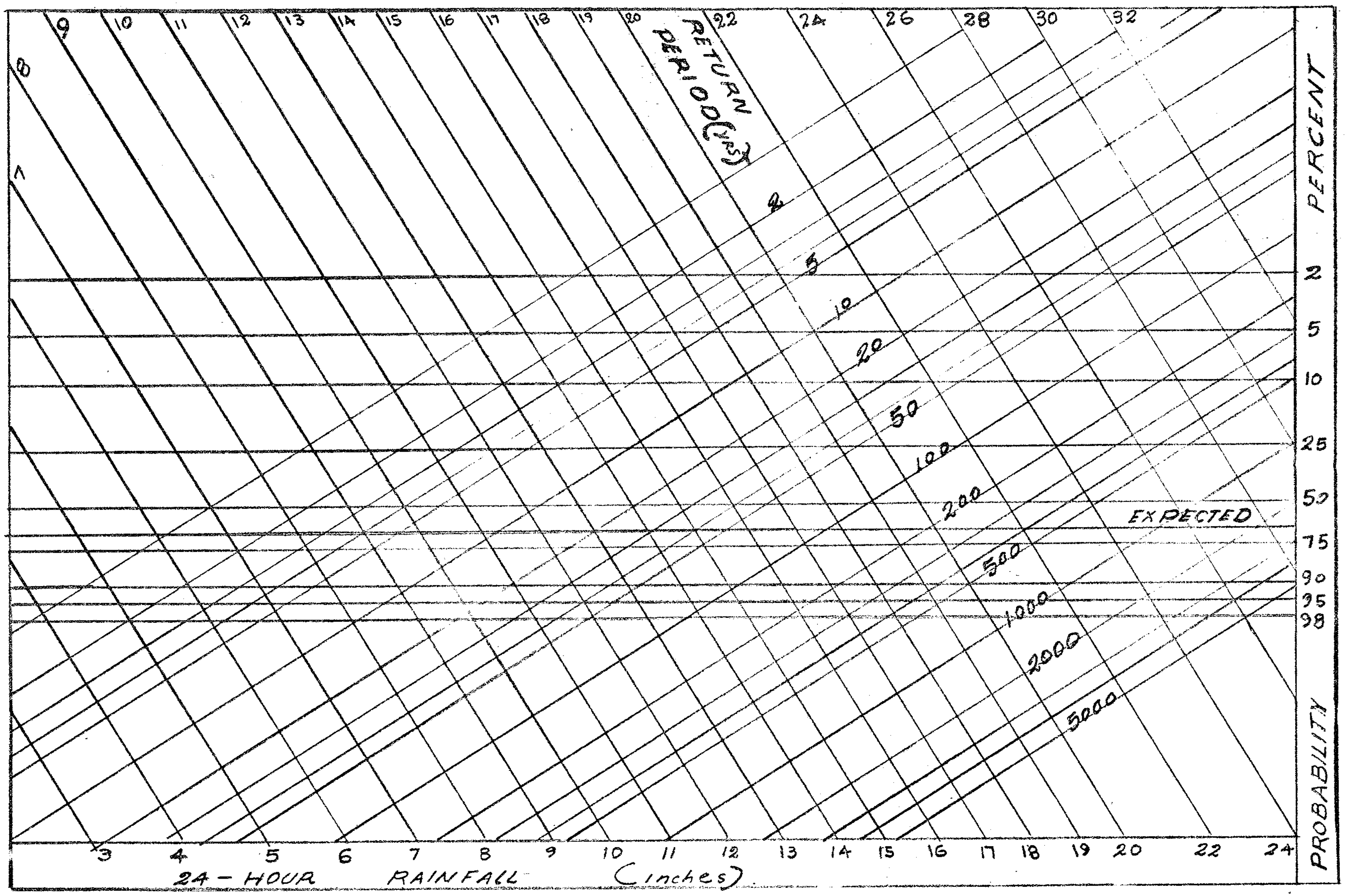

FIG 5 LAUTHALA BAY — NOMOGRAM FOR OBTAINING PROBABILITY OF SPECIFIED AMOUNTS OF DAILY RAINFALL IN SPECIFIED RETURN PERIODS